RSME 2023 LEÓN

VI CONGRESO DE JÓVENES INVESTIGADORES DE LA RSME

LEÓN 6-10 FEBRERO 2023

ORGANIZAN

universidad de león

Real Sociedad Matemática Española

COLABORAN

fgulem Fundación General Universidad de León y Empresa

Escuela de Ingenierías Industrial, Informática y Aeroespacial

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DIPUTACIÓN DE LEÓN

AYUNTAMIENTO DE LEÓN

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Bienvenida

Bienvenidos al VI Congreso de Jóvenes Investigadores de la Real Sociedad Matemática Española (RSME). Esta edición se celebra del 6 al 10 de febrero de 2023 en la Universidad de León.

Tras la buena acogida de las pasadas ediciones, el objetivo de este congreso sigue siendo el de ser un lugar de encuentro donde jóvenes investigadores en matemáticas puedan presentar sus trabajos más recientes, intercambiar ideas, conocer investigadores de otras universidades y establecer contactos que puedan ser productivos y perdurar en el tiempo.

Para favorecer estos objetivos, el programa cuenta con 7 charlas plenarias y 24 sesiones paralelas que abarcan distintas áreas de las matemáticas, así como una sesión de pósters.

Además del contenido puramente científico, en el congreso habrá otras actividades en las que se tratarán temas que preocupan a los jóvenes matemáticos. En la sesión de ANECA se hablará sobre los primeros pasos de la carrera docente e investigadora y su evaluación. Además, en las mesas redondas del congreso se debatirá sobre temas de interés para los asistentes y la sociedad en general, como las matemáticas en la educación y los distintos perfiles de matemáticos que realizan I+D+i. También abierta a la sociedad será la charla de divulgación Bayes y el dardo perdido, a cargo de Anabel Forte.

El programa se completa con una agenda social que incluye, además de la cena de gala, visitas culturales en la ciudad de León.

Esperamos que vuestra estancia en la Universidad de León y en nuestra ciudad sea fructífera en lo científico y muy grata en lo personal.

Comité organizador

Presidenta: Adriana Suárez Corona	*Universidad de León*
Noemí de Castro García	*Universidad de León*
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Mercedes Pelegrín García	*Fair Isaac Corporation*
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Abraham Rueda Zoca	*Universidad de Murcia*
Daniel Sanz Alonso	*University of Chicago*
Marithania Silvero Casanova	*Universidad de Sevilla*

Programa general

VI Congreso de Jóvenes Investigadores de la RSME					
	Lunes	Martes	Miércoles	Jueves	Viernes
8:30 - 9:00	**Registro** Hall de la Escuela de Ingenierías				
9:00 - 10:00	**Inauguración** Salón de actos	**Plenaria** V. Guerrero	**Plenaria** U. Etayo	**Plenaria** A. del Pino	**Plenaria** X. Fernández Real
10:00 - 11:00	**Plenaria** D. Sanz	**S. Paralelas**	**S. Paralelas**	**S. Paralelas**	**S. Paralelas**
11:00 - 12:00	**Café** Hall **Charla** ANECA	**Sesión de Pósters y Café** Hall	**Café** Hall		
12:00 - 13:00	**S. Paralelas**	**S. Paralelas**	**S. Paralelas**	**S. Paralelas**	**S. Paralelas**
13:00 - 14:00					**Clausura** Salón de Actos
14:00 - 15:30	**Comida** Cafetería Dani y José				
15:30 - 16:30	**Junta RSME** Salón de grados	**Plenaria** D. Beltrán	**Visita Guiada Grupo 1**	**Plenaria** A. Garrido	
16:30 - 17:30	**Mesa Redonda** Matemátic@s en educación	**Mesa Redonda** De profesión: matemático haciendo I+D+i		**Café** Hall **S. Paralelas**	
17:30 - 18:30	**Café** Hall	**Café** Hall	**Visita Guiada Grupo 2**		
18:30 - 20:00	**S. Paralelas**	**S. Paralelas**		**Charla** A. Forte	

☞ La cena de gala tendrá lugar el miércoles, en el Hotel Barceló Conde Luna, situado en la Avda. de la Independencia, 7, León.

Programa detallado

Lunes, 6 de febrero de 2023

8:30 - 9:00 Registro y recogida de documentación – Hall Principal

9:00 - 10:00 Acto de Bienvenida – Salón de Actos

10:00 - 11:00 Charla plenaria – Salón de Actos

> Daniel Sanz (University of Chicago), *Fundamentos Matemáticos del Aprendizaje Semi-Supervisado*
> Moderadora: María Ángeles García Ferrero

11:00 - 11:30 Café – Hall 2

11:30 - 12:30 Charla – Salón de Actos

> Clelia Martínez, Directora de la División de Evaluación de Profesorado de ANECA

12:30 - 13:30 Sesiones Paralelas L1

Geometría simpléctica y dinámica Hamiltoniana – Aula 201-A

Moderador: Robert Cardona

12:30 *Semi-analytical computation of center-stable and center-unstable manifolds in the geostationary belt*, Miquel Barcelona.

13:00 *La conjetura de Arnold para variedades b^m-simplécticas*, Joaquim Brugués.

Geometría Diferencial – Aula 207

Moderador: Erik Sarrion-Pedralva

12:30 *The Audibility problem from a non-compact point of view using generalized Heisenberg groups*, Jose Manuel Fernández-Barroso.

13:00 *High energy eigenfunctions on flat tori*, Alba García-Ruiz.

Geometría Algebraica I: Fundamentos – Aula 206

Moderador: Emilio Franco

12:30 *Fibrados principales, grupos no conexos y cubiertas*, Guillermo Barajas.

13:00 *La fibración de Hitchin: análogos y generalizaciones*, Guillermo Gallego.

Investigación Operativa – 217-B

Moderador: Víctor España

12:30 *Sustainable last-mile delivery using public transport*, Alberto Santini.

13:00 *A mathematical model for the Home Health Care Routing and Scheduling Problem*, Consuelo Parreño Torres.

Avances y aplicaciones de la Estadística – Aula 212

Moderador: Pablo Morala

12:30 *Integrated Depths for Partially Observed Functional Data*, Antonio Elías.

12:50 *Multivariate epigraph and hypograph indexes. Properties and applications*, Belén Pulido.

13:10 *A quantile based dimension reduction methodology for functional data*, Álvaro Méndez Civieta.

Mathematical challenges in fluid dynamics and kinetic theory – Aula 211

Moderador: David Poyato

12:30 *Monokineticity and mean-field limit for strongly singular Cucker-Smale model*, Michał Fabisiak.

13:00 *Homogenization of Stokes equation with randomly rotating particles*, Marta Leocata.

Análisis Matemático Aplicado y Ecuaciones Diferenciales – Aula 217-A

Moderador: Rafael Granero-Belinchón

12:30 *Uniform in gravity estimates for 2D water waves*, Siddhant Govardhan Agrawal.

13:00 *A limiting absorption principle for time-harmonic isotropic Maxwell and Dirac equation*, Lucrezia Cossetti.

13:30 - 15:30 Comida – Cafetería Dani & Jose

15:30 - 16:30 Junta RSME – Salón de Grados

16:30 - 17:45 Mesa redonda – Salón de Actos

Matemátic@s en educación

Moderadora: Itzíar García Honrado

Sara Álvarez Morán, Profesora de secundaria y profesora asociada de la Universidad de Oviedo.

Daniel Cao Labora, Ayudante Doctor de la Universidad de Santiago de Compostela.

Pablo Martín Álvarez, Profesor de secundaria y presidente de la asociación castellano-leonesa de educación matemática "Miguel de Guzmán"

Luis José Rodríguez Muñiz, Catedrático de la Universidad de Oviedo y Vicepresidente segundo de la RSME

17:45 - 18:15 Café – Hall 2

18:15 - 19:45 Sesiones paralelas L2

Geometría simpléctica y dinámica Hamiltoniana – Aula 201-A

Moderador: Robert Cardona

18:15 *Integrabilidad en sistemas hamiltonianos de contacto*, Asier López-Górdon.

Geometría Diferencial – Aula 207

Moderador: Erik Sarrion-Pedralva

18:15 *Prescribing values of a complete minimal surface whose Gauss map misses two point*, Ildefonso Castro-Infantes.

18:45 *Superficies de Bryant completas*, Jorge Hidalgo Calderón.

Teoría de Singularidades – Aula 205

Moderador: Patricio Almirón

18:15 *El Teorema de Comparación Logarítmica y la Euler-homogeneidad en divisores libres*, Abraham Del Valle Rodríguez

18:45 *Zeta functions, abelian orbifold resolutions of singularities and the geometry of curves in weighted projective planes*, Juan Viu Sos

19:15 *Computing a difficult invariant of holomorphic map germs*, Roberto Giménez Conejero

Geometría Algebraica I: Fundamentos – Aula 206

Moderador: Alfonso Zamora

18:15 *Variedades de caracteres de grupos de nudos tóricos*, Javier Martínez

18:55 *Geometría Sasakiana y superficies complejas*, Juan Rojo

Investigación Operativa – 217-B

Moderadora: Consuelo Parreño Torres

18:15 *The Hampered Traveling Salesman Problem with Neighbourhoods*, Carlos Valverde

18:45 *A Branch and Price for the Min-Max Close-Enough Arc Routing Problem*, Miguel Reula

19:15 *El problema del cartero rural con una flota de drones*, Paula Segura

Avances y aplicaciones de la Estadística – Aula 212

Moderadores: Álvaro Méndez Cievieta y Belén Pulido

18:15 *Selección de variables en Twin Support Vector Machines*, Sandra Benítez Peña

18:33 *Procedimientos de selección de covariables para modelos de regresión en alta dimensión*, Laura Freijeiro González

18:51 *On the selection of linear and non-linear features in high dimensional additive models*, Manuel Navarro-García

19:09 *Features importance in machine learning: a 2-dimensional approach*, Inmaculada Gutiérrez

19:27 *Evaluation and challenges of interpretability methods for feature interactions in neural networks*, Pablo Morala

Mathematical challenges in fluid dynamics and kinetic theory – Aula 211

Moderador: Eduardo García-Juarez

18:45 *On the Rayleigh-Taylor instability*, Björn Gebhard

19:15 *Long time behaviour of BGK models for heat transfer*, Josephine Evans

Análisis Matemático Aplicado y Ecuaciones Diferenciales – Aula 217-A

Moderadora: Martina Magliocca

18:45 *Traveling waves close to the Couette flow*, Daniel Lear

19:15 *On Variable Lebesgue Spaces and Navier-Stokes equations*, Gastón Vergara-Hermosilla

8:30 - 9:00 Registro y recogida de documentación – Hall Principal

9:00 - 10:00 Charla plenaria – Salón de Actos

> Vanesa Guerrero (Universidad Carlos III de Madrid), *Mathematical Optimization and Data Science: a back and forth relationship*
> Moderadora: Marina Leal

10:00 - 11:00 Sesiones Paralelas M1

Geometría simpléctica y dinámica Hamiltoniana – Aula 201-A

Moderadora: Mar Giralt

10:00 *Positive measure of effective quasi-periodic motion near a diophantine torus*, Gerard Farré

10:30 *Integración de foliaciones singulares usando caminos*, Alfonso Garmendia

Geometría convexa – Aula 201-B

Moderador: Bernardo González Merino

10:00 *Brunn-Minkowski inequality for θ-convolution bodies via Ball's bodies*, David Alonso Gutiérrez

10:30 *A unimodular theory of reduced and complete convex bodies*, Ansgar Freyer

Geometría Diferencial – Aula 207

Moderador: Alberto Rodríguez-Vázquez

10:00 *Superficies de curvatura lineal de Weingarten predeterminada*, Irene Ortiz

10:30 *Inestabilidad de Plateau-Rayleigh para λ-solitones*, Antonio Bueno

Teoría de Singularidades – Aula 205

Moderador: Abraham Del Valle Rodríguez

10:00 *Study of algebraic integrability of foliations on Hirzebruch surfaces. An algorithm of computation depending on the genus*, Elvira Pérez Callejo

10:30 *The Seshadri constant for a divisorial valuation*, Carlos Jesús Moreno Ávila

Geometría Algebraica I: Fundamentos – Aula 206

Moderador: Emilio Franco

10:00 *Embedding superconformal vertex algebras from Killing spinors*, Andoni De Arriba De La Hera

10:30 *Futaki invariants for the Hull-Strominger system*, Raúl González Molina

Investigación Operativa – 217-B

Moderador: Alberto Torrejón Valenzuela

10:00 *Features of the λ-cent-dian solution network*, Natividad González

10:30 *A new source connection problem*, Laura Davila Pena

Teoría de la Probabilidad – Aula 212

Moderador: Ignacio Montes

10:00 *Procesos de ramificación defectuosos en ambiente variable*, Carmen Minuesa

10:30 *Un paseo aleatorio entre modelos epidemiológicos*, Giulia Binotto

Análisis Matemático Aplicado y Ecuaciones Diferenciales – Aula 217-A

Moderador: Diego Alonso Orán

10:00 *Free boundary dynamics of gravity Stokes flow*, Elena Salguero

10:30 *Quasiperiodic solutions in the Euler equation*, Francisco Torres Lizaur

11:00 - 11:30 Café y sesión de pósters – Hall 2

11:30 - 13:30 Sesiones Paralelas M2

Geometría simpléctica y dinámica Hamiltoniana – Aula 201-A

Moderadora: Mar Giralt

11:30 *Parabolic ejection and collision orbits for the restricted planar circular three body problem*, José Lamas

12:00 *Holomorphic h–principles over Stein manifolds*, Guillermo Sánchez

12:30 *Periodic perturbation of a 3D conservative flow with 2D heteroclinic connection to saddle-foci*, Ainoa Murillo

13:00 *Introduction to Contact Topology*, Dahyana Farias

Geometría Diferencial – Aula 207

Moderador: Alberto Rodríguez-Vázquez

11:30 *Overdetermined elliptic problems in the sphere*, Jing Wu

12:00 *Métricas curvatura homogéneas críticas para funcionales cuadráticos de la curvatura*, Sandro Caeiro Oliveira

12:30 *Técnicas de análisis geométrico para un problema variacional en geometría (pseudo-)Finsler*, Fidel F. Villaseñor

13:00 *El problema de Zermelo y su aplicación a la modelización de incendios forestales*, Enrique Pendás Recondo

Álgebras no asociativas y aplicaciones – Aula 205

Moderadora: María Pilar Páez Guillán

11:30 *Nilpotent Poisson-type algebras*, Amir Fernández Ouaridi

12:00 *Ideales invariantes en álgebras de caminos de Leavitt*, Cristóbal Gil Canto

12:30 *Dimensión de Gelfand-Kirillov y entropía de álgebras filtradas*, Iván Ruiz Campos

13:00 *Gradings induced by nilpotent elements*, Guillermo Vera de Salas

Geometría Algebraica I: Fundamentos – Aula 206

Moderadora: Alicia Tocino

11:30 *Semimódulos de valoración sobre el anillo local de una rama singular con un par de Puiseux*, Patricio Almirón

12:10 *Continuantes de Euler en geometría de Poisson no conmutativa*, David Fernández

12:50 *Lax colimits of posets with structure sheaves: application to descent*, Javier Sánchez

Teoría de grafos extremales y aleatorios – Aula 201-B

Moderador: Alberto Espuny Díaz

11:30 *Resilience for loose Hamilton cycles*, Richard Lang

12:10 *Maximum running times for graph bootstrap percolation processes*, Patrick Morris

12:50 *Transversal cycle factors in multipartite graphs*, Amarja Kathapurkar

Investigación Operativa – 217-B

Moderador: Francisco Temprano

11:30 *The α-CVaR and other alternatives to deal with Uncertainty in the Hybridized Maximal Covering Location Problem*, Ricardo Gázquez

12:00 *On Collaboration in Multi-period Discrete Location*, Carmen Ana Domínguez Bravo

12:30 *Optimizing Treatment Allocation to Maximize the Health of a Population*, Alba Olivares

13:00 *Improving algorithm efficiency for fairness allocation in optimization*, Alberto Torrejón Valenzuela

Teoría de la Probabilidad – Aula 212

Moderador: Ignacio Montes

11:30 *On approximate validation of models: a Kolmogorov–Smirnov-based approach*, Hristo Inouzhe

12:00 *Coeficientes de asimetría estadística asintóticamente normales*, Raúl Pérez-Fernández

12:30 *Making data fair through optimal transport*, Paula Gordaliza

13:00 *Un análisis de robustez Bayesiana multivariante con enfoque en principios de primas*, Marta Sánchez-Sánchez

Mathematical challenges in fluid dynamics and kinetic theory – Aula 211

Moderadores: Jaemin Park y Claudia García

11:30 *Turbulence and convex integration*, Francisco Mengual

12:00 *Macroscopic limit of the Vlasov-Poisson-Fokker-Planck model*, Alain Blaustein

12:30 *Study of the asymptotic behavior of the NNLIF model*, Alejandro Ramos Lora

13:00 *Highest Cusped Waves for the Burgers-Hilbert and Fractional KdV equations*, Joel Dahne

Análisis Matemático Aplicado y Ecuaciones Diferenciales – Aula 217-A

Moderadores: Diego Alonso Orán y Daniel Lear

11:30 *Rigorous thin film approximations of the one-phase unstable Muskat problem*, Edoardo Bocchi

12:00 *Spectral Analysis in Quantum Mechanics*, Naiara Arrizabalaga

12:30 *Keller estimates of the eigenvalues in the gap of Dirac operators*, Fabio Pizzichillo

13:00 *On the symmetry conjecture for eigenfunctions*, Ángel David Martínez Martínez

13:30 - 15:30 Comida – Cafetería Dani & Jose

15:30 - 16:30 Charla plenaria – Salón de Actos

> David Beltrán (Universitat de València), *Maximal averages along variable manifolds*
> Moderador: Guillermo Rey

16:30 - 17:45 Mesa redonda – Salón de Actos

De profesión: matemáticos haciendo I+D+i

Moderador: Marcos Gómez

Alfonso D. Bláquez, Freelance the datawarlock.

Covadonga Fernández, Banco Santander. Head of Credit Risk Model Validation.

Almudena Sastre Noriega, Team leader en el equipo SAP en NTT Data Valladolid.

Laura Toribio, Postdoctoral Researcher en CIEMAT (Centro de Investigaciones Energéticas, Medioambientales y Tecnológicas)

17:45 - 18:15 Café – Hall 2

18:15 - 19:45 Sesiones paralelas M3

Geometría simpléctica y dinámica Hamiltoniana – Aula 201-A

Moderadora: Mar Giralt

18:15 *An overview about bracket-generating distributions*, Javier Martínez-Aguinaga

Geometría convexa – Aula 201-B

Moderador: Antonio Cañete

18:15 *Large signed subset sums*, Bernardo González Merino

18:45 *Geometric inequalities involving different diameter definitions*, Mia Runge

19:15 *Fórmulas tubulares*, Juan Andrés Trillo

Geometría Diferencial – Aula 207

Moderador: Erik Sarrion-Pedralva

18:15 *Acciones de grupos finitos y nilvariedades: propiedad Jordan y rigidez*, Jordi Daura Serrano

18:45 *Desigualdad de tipo Basmajian para superficies Riemannianas con frontera*, David Fisac Camara

Teoría de Singularidades – Aula 205

Moderadora: Elvira Pérez Callejo

18:15 *Singularidades simples de foliaciones holomorfas de codimensión uno*, Beatriz Molina-Samper

18:45 *Interpretación geométrica de la base estándar de una curva plana*, David Senovilla Sanz

19:15 *Configuración local de ciclos de campos de vectores analíticos en* $\mathbb{R}^3$, María Martín Vega

Geometría Algebraica I: Fundamentos – Aula 206

Moderador: Alfonso Zamora

18:15 *Condition-based Low-Degree Approximation of Real Polynomial Systems*, Josué Tonelli-Cueto

18:55 *Esquemas exponenciables no conmutativos en geometría algebraica plana*, Julia Ramos González

Investigación Operativa – 217-B

Moderadora: Laura Davila Pena

18:15 *Optimización de la asignación de atraques considerando el tiempo de movimiento de las grúas*, Juan Francisco Correcher

18:40 *Estimating Ensemble weights for Bagging Regressors based on the Mean-Variance portfolio framework*, Thomas Ashley

19:00 *Single-linkage clustering with feature selection: a mathematical optimization approach*, Marina Leal

19:20 *A Mathematical Programming approach to the Minimum Normalized Cut Problem*, Francisco Temprano

Teoría de la Probabilidad – Aula 212

Moderador: Ignacio Montes

18:15 *Conjuntos credales representables mediante intervalos de probabilidad alcanzables y funciones de creencia*, Serafín Moral-García

18:45 *Determinando si un conjunto aleatorio es conexo*, Juan-Jesús Salamanca

19:15 *EM for Approximating Unidentifiable Counterfactual Queries*, Rafael Cabañas

Mathematical challenges in fluid dynamics and kinetic theory – Aula 211

Moderadora: Claudia García

18:45 *Small scale creation for the 2D Boussinesq Equation*, Jaemin Park

19:15 *Skeleton for the one-dimensional aggregation equation*, Juan Carlos Cantero

8:30 - 9:00 Registro y recogida de documentación – Hall Principal

9:00 - 10:00 Charla plenaria – Salón de Actos

> Ujué Etayo (Universidad de Cantabria), *¿Cómo distribuir puntos en la esfera?*
> Moderador: Josué Tonelli

10:00 - 11:00 Sesiones Paralelas X1

Geometría simpléctica y dinámica Hamiltoniana – Aula 201-A

Moderador: Cédric Oms

10:00 *Bohr-Sommerfeld quantization of b-symplectic toric manifolds*, Pau Mir

10:30 *Splitting of separatrices for rapidly forced pendulum with a perturbation without first harmonic*, Román Moreno

Geometría convexa – Aula 201-B

Moderador: Eduardo Lucas

10:00 *Prescribed mean curvature graphs in sub-Finsler Heisenberg groups*, Julián Pozuelo

10:30 *Proximity and flatness bounds via a volume argument*, Stefan Kuhlmann

Geometría Diferencial – Aula 207

Moderadora: Adela Latorre

10:00 *Estructuras geométricas en el fibrado tangente generalizado*, Pablo Gómez-Nicolás

10:30 *Futaki invariants for the Hull-Strominger system*, Raul Gonzalez Molina

Álgebras no asociativas y aplicaciones – Aula 205

Moderadora: Alejandra Sarina Córdova Martínez

10:00 *Natural families in evolution algebras*, Yolanda Cabrera Casado

10:30 *Grupos de automorfismos de las álgebras de evolución de Cayley*, Alicia Tocino Sánchez

Investigación Operativa – 217-B

Moderadora: Paula Segura

10:00 *The Boosted DC Algorithm for linearly constrained DC programming*, Rubén Campoy

10:30 *Un modelo aditivo basado en funciones lineales a trozos y aprendizaje automático para la estimación de funciones de producción*, Víctor España

Inferencia estadística frecuentista y aplicaciones – Aula 212

Moderador: José Ameijeiras Alonso

10:00 *Métodos no paramétricos para comparar curvas ROC con covariables*, Arís Fanjul Hevia

10:20 *La correlación de distancias. Aplicación en Análisis de Supervivencia*, María Vidal García

10:40 *Un nuevo contraste de multimodalidad para datos circulares*, Diego Bolón Rodríguez

Análisis Armónico y Teoría de Operadores – Aula 217-A

Moderador: Israel Rivera

10:00 *Fractional discrete Laplacian and (global) unique continuation*, Aingeru Fernández

10:30 *Characterization of the discrete Besov spaces by means of semigroups associated to the discrete Laplacian*, Alejandro Mahillo

Mathematical challenges in fluid dynamics and kinetic theory – Aula 211

Moderador: Diego Alonso Orán

10:00 *Existence and decay for some fourth order problems*, Martina Magliocca

10:30 *Muskat equation with unbounded data*, Omar Sanchez Antonio

Recent advances in local and nonlocal PDE's – Aula 206

Moderador: Ángel Arroyo

10:00 *Concentration versus Simplification in Aggregation-Diffusion Equations*, David Gómez-Castro

10:30 *Concentration phenomena arising in Aggregation Fast-Diffusion equations*, Alejandro Fernández-Jiménez

11:00 - 11:30 Café – Hall 2

11:30 - 13:30 Sesiones Paralelas X2

Geometría simpléctica y dinámica Hamiltoniana – Aula 201-A

Moderador: Cédric Oms

11:30 *Hamiltonian facets of classical gauge theories on E-manifolds*, Pablo Nicolás

12:00 *A degenerate Arnold Diffusion Mechanism in the Restricted 3 Body Problem*, Jaime Paradela

12:30 *A simple proof of Novikov's theorem*, Samuel Ranz

13:00 *Unique ergodicity of the horocyclic flow of surfaces without conjugate points*, Sergi Burniol

Geometría convexa – Aula 201-B

Moderador: David Alonso-Gutiérrez

11:30 *Discrete variations of the Brunn-Minkowski inequality*, Eduardo Lucas

11:55 *Ellipsoids in convex bodies of a given asymmetry*, Florian Grundbacher

12:20 *A discrete approach to Zhang's inequality*, Javier Martín Goñi

12:45 *Affine subspace concentration conditions for centered polytopes*, Christian Kipp

13:10 *The Cheeger set of a rotationally symmetric planar convex body*, Antonio Cañete

Geometría Diferencial – Aula 207

Moderadora: Adela Latorre

11:30 *Solitones de Ricci homogéneos y grupos nilpotentes de Iwasawa*, Víctor Sanmartín-López

12:00 *Upper and lower bounds on the filling radius*, A. Manuel Cuerno

12:30 *Quantizability of manifold invariants*, Ángel González-Prieto

Álgebras no asociativas y aplicaciones – Aula 205

Moderador: Xabier García Martínez

11:30 *On metric generalized Jordan superpairs*, Diego Aranda Orna

12:00 *Categorías tensoriales, álgebras y superálgebras*, Alberto Daza García

12:30 *Local quadratic Lie algebras*, Jorge Roldán López

13:00 *Divisibilidad en álgebras Hom-asociativas*, Germán García Butenegro

Teoría de grafos extremales y aleatorios – Aula 212

Moderador: Alberto Espuny Díaz

11:30 *How to build a pillar: a proof of Thomassen's conjecture*, Irene Gil Fernández

12:10 *Hypergraphs with minimum uniform Turán density*, Ander Lamaison

12:50 *Ramsey numbers with prescribed rate of growth*, Simón Piga

Análisis Funcional – Aula 217-B

Moderador: Christian Cobollo

11:30 *On the numerical index of the real two-dimensional L_p space*, Alicia Quero de la Rosa

12:00 *Banach spaces contained in the set of strongly norm-attaining Lipschitz mappings*, Óscar Roldán Blay

13:00 *La convolución y la propiedad de la diferenciabilidad*, Pablo Jiménez Rodríguez

Análisis Armónico y Teoría de Operadores – Aula 217-A

Moderador: Aingeru Fernández

11:30 *Weighted estimates on the infinite rooted k-ary tree*, Israel Rivera

12:00 *Resultados en extrapolación para operadores bilineales*, Teresa Luque

12:30 *Extrapolation via Sawyer-type inequalities*, Eduard Roure

13:00 *Extrapolation in quasi-Banach function spaces*, Zoe Nieraeth

Recent advances in local and nonlocal PDE's – Aula 206

Moderador: Ángel Arroyo

11:30 *A sharp Hardy inequality in fractional Sobolev spaces*, Francesca Bianchi

12:00 *Multiple solutions for the nonlocal Liouville equation in $\mathbb{R}$*, Antonio J. Fernández

12:30 *Bound and ground states for a family of singular systems in $\mathbb{R}^N$*, Rafael López-Soriano

13:00 *Bernstein method for nonlocal operators*, Marvin Weidner

13:30 - 15:30 Comida – Cafetería Dani & Jose

16:00 - 17:30 Visita guiada Grupo I

17:30 - 19:00 Visita guiada Grupo II

21:00 Cena de gala – Hotel Barceló Conde Luna

8:30 - 9:00 Registro y recogida de documentación – Hall Principal

9:00 - 10:00 Charla plenaria – Salón de Actos

> Álvaro del Pino Gómez (Universiteit Utrecht), *Relaciones diferenciales no-locales*
> Moderador: Francisco Javier Torres de Lizaur

10:00 - 11:00 Sesiones Paralelas J1

Differential Geometry, Mathematical Physics and Control Theory – Aula 207

Moderadores: Asier López Gordón y Xavier Rivas Guijarro

10:00 *Shannon information entropy for the Darboux III quantum nonlinear oscillator*, Iván Gutiérrez Sagredo

10:30 *Periodic orbits and Birkhoff sections of stable Hamiltonian structures*, Robert Cardona

Geometría Algebraica II: Aplicaciones y Computación – Aula 201-A

Moderadores: Josué Tonelli-Cueto y Angélica Torres

10:00 *Algebraic reliability of multi-state systems*, Patricia Pascual-Ortigosa

10:30 *Average degree of the essential variety!*, Elima Shehu

Matemáticas en la Sociedad de la Información – Aula 211

Moderadora: Verónica Requena

10:00 *Self-dual Convolutional Codes*, Julia Lieb

10:30 *Nuevas ideas para mejorar la seguridad de un criptosistema de tipo McEliece basado en códigos convolucionales*, Miguel Beltrá

Inferencia estadística frecuentista y aplicaciones – Aula 201-B

Moderadora: María Alonso Pena

10:00 *Probability of default estimation using mixture cure models*, Rebeca Peláez Suárez

10:20 *Testing a linear-circular parametric regression model*, Andrea Meilán Vila

10:40 *The FMM approach for mathematical modelling of the 24-h cardiac rhythm*, Yolanda Larriba

Análisis Funcional – Aula 217-B

Moderadora: Alicia Quero de la Rosa

10:00 *Sumas torcidas de espacios $c_0(I)$*, Alberto Salguero Alarcón

10:30 *Derivación de funtores en espacios de Banach*, Nazaret Trejo Arroyo

Análisis Armónico y Teoría de Operadores – Aula 217-A

Moderador: Adrián González Pérez

10:00 *The problem of thermal noise in quantum double models*, Antonio Perez Hernández

10:30 *Greedy approximation algorithms for sparse collections*, Guillermo Rey

Sistemas Dinámicos y Ecuaciones Diferenciales – Aula 212

Moderadora: Cristina Lois Prados

10:00 *Attractor for non-autonomous Lotka-Volterra cooperative systems*, Juan García-Fuentes

10:30 *Contributions to the classification of the global dynamics of polynomial differential systems*, Érika Diz-Pita

Recent advances in local and nonlocal PDE's – Aula 206

Moderador: Juan-Carlos Felipe-Navarro

10:00 *Nonlocal concave-convex critical problems with mixed boundary conditions*, Alejandro Ortega

10:30 *Positive solutions to critical competitive systems in low dimension*, María Medina

Avances recientes en Teoría de Control – Aula 205

Moderador: Diego A. Souza

10:00 *Uncertainty principles in control theory of PDEs*, Ivan Moyano

10:30 *An optimal boundary control problem related to the Navier-Stokes equations in dimension 2*, Irene Marín-Gayte

11:00 - 11:30 Café – Hall 2

11:30 - 13:30 Sesiones Paralelas J2

Topología – Aula 201-B

Moderador: Mario Fuentes

11:30 *Derivadores y homotopía: el cálculo de límites*, Beatriz Álvarez

12:00 *Cohomología de haces para CL-shellable posets*, Guille Carrión

12:30 *Estabilidad de Sucesiones Espectrales de Mayer–Vietoris*, Álvaro Torras

13:00 *Descomposiciones simpliciales y campos de vectores discretos*, Juan Delgado

Differential Geometry, Mathematical Physics and Control Theory – Aula 207

Moderadores: Asier López Gordón y Xavier Rivas Guijarro

11:30 *Reduction by symmetries and applications to optimal control*, Leonardo J. Colombo

12:15 *Nonholonomic systems and the inverse problem*, Marta Farré Puiggalí

12:45 *An introduction to contact Hamiltonian systems*, Manuel Lainz

Geometría Algebraica II: Aplicaciones y Computación – Aula 201-A

Moderadores: Josué Tonelli-Cueto y Angélica Torres

11:30 *Context-Specific Decomposable Models*, Eliana Duarte

12:00 *Mu-basis of certain parametrizations in two variables*, Amrutha Balachandran Nair

12:30 *Algebraic Geometry in Optimization*, Lukas Gustafsson

13:00 *In search of energy minimizers on the Grassmannian*, Pedro R. López-Gómez

Matemáticas en la Sociedad de la Información – Aula 211

Moderador: Carlos Vela

11:30 *Bounds for flag codes*, Miguel Ángel Navarro Pérez

12:00 *Subfield subcodes of projective Reed-Muller Codes*, Rodrigo San José Rubio

12:30 *Permutation Decoding in Group Codes*, Fabián Ricardo Molina Gómez

13:00 *Quantum codes defined by trace-depending polynomials*, Helena Martín Cruz

Análisis Funcional – Aula 217-B

Moderadores: Alberto Salguero Alarcón

11:30 *El problema del subespacio complementado en retículos de Banach*, David de Hevia Rodríguez

12:00 *Dualidad en retículos de Banach libres*, Enrique Sánchez Pérez

12:30 *Funciones uniformemente convexas y teorema de Enflo*, Guillaume Grelier

13:00 *Geometría local vs geometría global en la esfera unidad de un espacio de Banach*, Andrés Quilis

Análisis Armónico y Teoría de Operadores – Aula 217-A

Moderadores: Antonio Pérez Hernández y Odí Soler

11:30 *Matrix-valued Calderón-Zygmund theory on noncommutative Hardy spaces*, Antonio Ismael Cano

12:00 *Hilbert transforms for groups acting on tree-like structures*, Adrián González Pérez

12:30 *Group Equivariant Machine Learning*, María Teresa Arias

13:00 *Pointwise ergodic theorems on nilpotent groups*, Tomasz Szarek

Sistemas Dinámicos y Ecuaciones Diferenciales – Aula 212

Moderadores: Cristina Lois Prados y José Manuel Uzal Couselo

11:30 *Tipping phenomena on nonautonomous single-species population models subject to Allee effect*, Jesús Dueñas

12:00 *Planar Hamiltonian systems: Index theory and applications*, Eduardo Muñoz-Hernández

12:30 *Applications for L_3 in celestial mechanics: from mathematical modelling to real-system comparisons*, Begoña Nicolás

13:00 *The Hydrogen atom in a circularly polarized microwave field: Trajectories from and to the nucleus*, Óscar Rodríguez

Aspectos computacionales y aplicaciones de las ecuaciones en derivadas parciales hiperbólicas – Aula 205

Moderadores: Celia Caballero Cárdenas y Ernesto Pimentel García

11:30 *High-order Implicit-Explicit scheme for the Exner model without gravity effects*, Emanuele Macca

11:50 *Exactly well-balanced schemes for the shallow-water equations with Coriolis forces*, Víctor González Tabernero

12:10 *Una familia de esquemas de volúmenes finitos implícitos y semi-implícitos de alto orden bien equilibrados para sistemas de leyes de equilibrio unidimensionales*, Irene Gómez Bueno

12:30 *Assessing the performance of well-balanced WENO and TENO schemes for atmospheric flows*, Adrián Navas Montilla

12:50 *Esquemas de volúmenes finitos implícitos y semi-implícitos bien equilibrados de tipo Lagrangiano-Proyectado para las ecuaciones de aguas someras*, Celia Caballero Cárdenas

13:10 *Reduced-order models applied to hyperbolic transport equations*, Pablo Solán Fustero

Recent advances in local and nonlocal PDE's – Aula 206

Moderador: Juan-Carlos Felipe-Navarro

11:30 *Self-similar solutions for the generalized surface quasi-geostrophic equation*, Claudia García

12:00 *An interior regularity result for the MEMS problem*, Renzo Bruera

12:30 *Global regularity of stable solutions to semilinear equations*, Íñigo U. Erneta

13:00 *A new class of double phase variable exponent problems and a Nehari manifold approach*, Ángel Crespo-Blanco

13:30 - 15:30 Comida – Cafetería Dani & Jose

15:30 - 16:30 Charla plenaria – Salón de Actos

> Alejandra Garrido (Universidad Autónoma de Madrid – ICMAT), *Un paseo por el bosque: árboles y sus simetrías*
> Moderadora: Oihana Garayalde

16:30 - 17:00 Café – Hall 2

17:00 - 19:00 Sesiones paralelas J3

Topología – Aula 201-B

Moderador: Mario Fuentes

17:00 *Un ejemplo "elemental" de cuantización por deformación con Álgebras de Factorización*, Víctor Carmona

17:30 *Computing signatures of nearby fibres of parametrized complex surfaces*, Roberto Giménez

18:00 *Grupo de cobordismos Lagrangianos*, Álvaro Muñiz

18:30 *Una introducción (muy introductoria) a la (co)homología de intersección*, Alba Sedón

Differential Geometry, Mathematical Physics and Control Theory – Aula 207

Moderadores: Xavier Rivas Guijarro y Miguel Ángel Berbel López

17:00 *Noether Symmetries in Classical Field Theory and Pre-multisymplectic Geometry*, Arnoldo Guerra IV

17:30 *Contracting Forced Lagrangian and Contact Lagrangian Systems: application to nonholonomic systems with symmetries*, Alexandre Anahory Simoes

18:00 *Variational integrators for reduced Lagrange–Dirac systems*, Álvaro Rodríguez Abella

18:30 *A polycosymplectic reduction*, Bartosz Maciej Zawora

Teoría de Grupos – Aula 205

Moderadores: María Cumplido e Iván Chércoles

17:00 *A Kurosh-type theorem for Lie algebras*, Simone Blumer

17:20 *Cohomological uniqueness of finite groups*, Lander Guerrero

17:40 *Hypercubical groups: a trip from RAAGs to Borromean rings*, Alberto Cassella

18:00 *Abelian splittings of pro-p Right-angled Artin groups*, Matteo Pintonello

18:20 *Rigidez finita de complejos de curvas no separantes*, Rodrigo de Pool

Geometría Algebraica II: Aplicaciones y Computación – Aula 201-A

Moderadores: Josué Tonelli-Cueto y Angélica Torres

17:00 *Decomposing minimal problems in computer vision for 3D reconstruction using Galois/monodromy groups*, Margaret Regan

17:30 *Lines in Algebraic Vision*, Felix Rydell

18:00 *Birational Free Form Deformations of degree* $1 \times 1 \times 1$, Pablo Gonzalez Mazon

18:30 *Toric Sylvester forms and applications*, Carles Checa

Teoría de grafos extremales y aleatorios – Aula 212

Moderador: Alberto Espuny Díaz

17:00 *Preservation theorems on random graphs*, Alberto Larrauri

17:40 *The expected number of perfect matchings in cubic planar graphs*, Clément Requilé

Matemáticas en la Sociedad de la Información – Aula 211

Moderador: Miguel Ángel Navarro

17:00 *On the possible sequences of generalized weights of linear codes*, Umberto Martínez Peñas

17:30 *Gray maps and linearities*, Carlos Vela

18:00 *Simetrías gauge para preservación de la privacidad aprendizaje automático*, Alex Pozas-Kerstjens

18:30 *Rank metric codes over Galois rings*, Markel Epelde

Análisis Funcional – Aula 217-B

Moderadora: Alicia Quero de la Rosa

17:00 *Convergence and spaceability within the disc algebra*, Pablo José Gerlach Mena

17:30 *Holomorphic functions attaining their weighted norm*, Rubén Medina Sabino

18:00 *Phelps Property U and $C(K)$ spaces*, Christian Cobollo Gómez

Análisis Armónico y Teoría de Operadores – Aula 217-A

Moderador: Tomasz Szarek

17:00 *Fine spectrum of weighted averaging operators of hyperbolic type*, Jesús Oliva

17:30 *Weak spectral decompositions and invariant subspaces*, Miguel Monsalve

18:00 *Finite rank perturbations of normal operators: invariant subspaces and decomposability*, Javier González Doña

Recent advances in local and nonlocal PDE's – Aula 206

Moderador: Tomás Sanz-Perela

17:00 *Evolution driven by the infinity fractional Laplacian*, Félix del Teso

17:30 *The KPP diffusion-reaction problem in the hyperbolic space*, Irene Gonzálvez

18:00 *Blowing up solutions to elliptic problems with natural growth in the gradient*, Salvador López

18:30 *Generic regularity of free boundaries for the thin obstacle problem*, Damià Torres

19:00 - 20:00 Charla: Bayes y el dado perdido. Anabel Forte (Universitat de València) – Salón de Actos

8:30 - 9:00 Registro y recogida de documentación – Hall Principal

9:00 - 10:00 Charla plenaria – Salón de Actos

> Xabier Fernández-Real (École Polytechnique Fédérale de Lausanne), *The fractional obstacle problem*
> Moderador: Juan Carlos Felipe Navarro

10:00 - 11:00 Sesiones Paralelas V1

Topología – Aula 201-B

Moderador: Sergio García

10:00 *Apollonian packings and knots*, Iván Rasskin

10:30 *Homología de Khovanov y enlaces ligados*, O'Bryan Cárdenas

Teoría de Grupos – Aula 205

Moderadores: María Cumplido e Iván Chércoles

10:00 *Hausdorff dimension of groups acting on trees*, Marialaura Noce

10:30 *Structure of Finitely Presented Bestvina-Brady Groups*, Mallika Roy

Matemáticas en la Sociedad de la Información – Aula 211

Moderador: Daniel Sadornil

10:00 *Functional Commitments for Circuits from Falsifiable Assumptions*, David Balbás

10:30 *Generación eficiente de números con propiedades de ruido gaussiano*, Francisco Javier Soto Sánchez

Inferencia estadística frecuentista y aplicaciones – Aula 207

Moderadora: Yolanda Larriba

10:00 *A broad approach for kernel regression with circular covariates*, María Alonso Pena

10:20 *Survival analysis with a cure fraction and multidimensional covariates*, Beatriz Piñeiro Lamas

Análisis Armónico y Teoría de Operadores – Aula 217-A

Moderadores: Odí Soler y Miguel Monsalve

10:00 *Un modelo funcional en el anillo*, Glenier Bello

10:30 *Characterising multiparameter dyadic BMO spaces with commutators*, Odí Soler

Sistemas Dinámicos y Ecuaciones Diferenciales – Aula 212

Moderador: José Manuel Uzal Couselo

10:00 *Bursting neurons: bifurcation structure of spike-adding processes in the Hindmarsh-Rose model*, Lucía Pérez

10:30 *Invariant manifolds of vector fields with parabolic invariant tori*, Clara Cufí-Cabré

Aspectos computacionales y aplicaciones de las ecuaciones en derivadas parciales hiperbólicas – Aula 206

Moderadora: Irene Gómez Bueno

10:00 *Numerical evolution of the resistive relativistic magnetohydrodynamic equations: a minimally implicit Runge-Kutta scheme*, Samuel Santos Pérez

10:20 *Vertical discretizations in depth-averaged models for geophysical flows*, José Garres Díaz

10:40 *High-order fully well-balanced numerical methods for one-dimensional blood flow with discontinuous properties*, Ernesto Pimentel García

Avances recientes en Teoría de Control – Aula 217-B

Moderador: Diego A. Souza

10:00 *On the global controllability of the viscous Boussinesq system*, Kévin Le Balc'h

10:30 *Insensitizing control for the Boussinesq system using Carleman estimates*, Jefferson Prada

11:00 - 11:30 Café – Hall 2

11:30 - 13:00 Sesiones Paralelas V2

Topología – Aula 201-B

Moderador: Sergio García

11:30 *The cohomological index of free Z/p-actions*, Rafael Gomes

12:00 *Distancia homotópica entre aplicaciones*, David Mosquera

12:30 *Homotopía racional parametrizada estable con modelos de Lie*, Alejandro Saiz

Teoría de Grupos – Aula 205

Moderadores: María Cumplido e Iván Chércoles

11:30 *Cohomology and fusion systems*, Oihana Garayalde

12:00 *Core-portraits in self-similar groups*, Jone Uria

12:30 *Automorphism groups of Cayley graphs of Coxeter groups*, Federico Berlai

Matemáticas en la Sociedad de la Información – Aula 211

Moderador: Miguel Beltrá

11:30 *A generalized Euclidean algorithm for the multisequence Skew-Feedback Shift-Register synthesis problem*, José Manuel Muñoz

12:00 *La transformación de Fujisaki-Okamoto y su relevancia en algoritmos post-cuánticos*, Miguel Ángel González de la Torre

12:30 *Design of a Post-Quantum Group Authenticated Key Exchange protocol using Classic McEliece, Kyber, NTRU, and Saber*, Misael Enrique Marriaga Castillo

Análisis Armónico y Teoría de Operadores – Aula 217-A

Moderador: Miguel Monsalve

11:30 *Trace of Bloch and BMO(D) functions on Interpolating Sequences*, Alberto Dayan

12:00 *Carleson measure type problem for the tent spaces and integration operators*, Tanausú Aguilar

12:30 *Operators induced by radial measures acting on the Dirichlet space*, Alejandro Más

Sistemas Dinámicos y Ecuaciones Diferenciales – Aula 212

Moderadora: Cristina Lois Prados

11:30 *Structure of attractors for a nonlocal Chafee-Infante problem*, Rubén Caballero

12:00 *A link between difference equations and delay differential equations*, Sebastián Buedo-Fernández

Avances recientes en Teoría de Control – Aula 217-B

Moderador: Jon Asier Bárcena Petisco

11:30 *Game theory in mathematical ecology*, Domènec Ruiz i Balet

12:00 *Control of advection-diffusion equations on networks and singular limits*, Nicola de Nitti

13:00 - 13:30 Clausura del congreso – Salón de Actos

13:30 - 15:30 Comida – Cafetería Dani & Jose

Calendario detallado

LUNES, 6 de febrero								
8:30 - 9:00	Recogida de documentación- Hall principal							
9:00 - 10:00	Acto de bienvenida - Salón de Actos							
10:00 - 11:00	Charla plenaria: Salón de Actos *Fundamentos Matemáticos del Aprendizaje Semi-Supervisado*, Daniel Sanz Alonso							
11:00 - 11:30	Café - Hall 2							
11:30 - 12:30	Charla: Clelia Martínez (ANECA) - Salón de Actos							
Aula	201-A	205	206	207	211	212	217-A	217-B
12:30 - 13:30	Geometría simpléc. y dinám. Hamilton.		Geom. alg. I	Geom. dif.	Math. ch. in fluid dyn. and kinetic th.	Av. y apl. estadística	Análisis matemático apl. y ED	IO
13:30 - 15:30	Comida - Cafetería Dani y José							
15:30 - 16:30	Junta de la RSME - Salón de Grados							
16:30 - 17:45	Mesa redonda: Matemátic@s en educación Salón de Actos							
17:45 - 18:15	Café - Hall 2							
Aula	201-A	205	206	207	211	212	217-A	217-B
18:15 - 19:45	Geometría simpléc. y dinám. Hamilton.	T. singul.	Geom. alg. I	Geom. dif.	Math. ch. in fluid dyn. and kinetic th.	Av. y apl. estadística	Análisis matemático apl. y ED	IO

MARTES, 7 de febrero									
9:00 - 10:00	Charla plenaria: Salón de Actos *Mathematical Optimization and Data Science: a back and forth relationship*, Vanesa Guerrero								
Aula	201-A	201-B	205	206	207	211	212	217-A	217-B
10:00 - 11:00	Geometría simpléc. y dinám. Hamilton.	Geom. convexa	T. singul.	Geom. alg. I	Geom. dif.		T. probabilidad	Análisis matemático apl. y ED	IO
11:00 - 11:30	Café y Sesión de Posters - Hall 2								
Aula	201-A	201-B	205	206	207	211	212	217-A	217-B
11:30 - 13:30	Geometría simpléc. y dinám. Hamilton.	T. grafos extremal. y aleat.	Álg. no asociativ. y apl.	Geom. alg. I	Geom. dif.	Math. ch. in fluid dyn. and kinetic th.	T. probabilidad	Análisis matemático apl. y ED	IO
13:30 - 15:30	Comida - Cafetería Dani y José								
15:30 - 16:30	Charla plenaria: Salón de Actos *Maximal averages along variable manifolds*, David Beltrán Portalés								
16:30 - 17:45	Mesa redonda: "De profesión: Matemáticos haciendo I+D+i" Salón de Actos								
17:45 - 18:15	Café - Hall 2								
Aula	201-A	201-B	205	206	207	211	212	217-A	217-B
18:15 - 19:45	Geometría simpléc. y dinám. Hamilton.	Geom. convexa	T. singul.	Geom. alg. I	Geom. dif.	Math. ch. in fluid dyn. and kinetic th.	T. probabilidad		IO

MIÉRCOLES, 8 de febrero									
9:00 - 10:00	Charla plenaria: Salón de Actos *¿Cómo distribuir puntos en la esfera?*, Ujué Etayo Rodríguez								
Aula	201-A	201-B	205	206	207	211	212	217-A	217-B
10:00 - 11:00	Geometría simpléc. y dinám. Hamilton.	Geom. convexa	Álg. no asociativ. y apl.	Rec. adv. local and nonlocal PDEs	Geom. dif.	Math. ch. in fluid dyn. and kinetic th.	Inf. est. frec. y apl.	Análisis armónico y t. operadores	IO
11:00 - 11:30	Café - Hall 2								
Aula	201-A	201-B	205	206	207	211	212	217-A	217-B
11:30 - 13:30	Geometría simpléc. y dinám. Hamilton.	Geom. convexa	Álg. no asociativ. y apl.	Rec. adv. local and nonlocal PDEs	Geom. dif.		T. grafos extremal. y aleat.	Análisis armónico y t. operadores	Análisis funcional
13:30 - 15:30	Comida - Cafetería Dani y José								
Tarde	Visita guiada. Grupo I: 16:00-17:30. Grupo II: 17:30-19:00								
21:00	Cena de gala - Hotel Barceló Conde Luna								

JUEVES, 9 de febrero									
9:00 - 10:00	Charla plenaria: Salón de Actos *Relaciones diferenciales no-locales*, Álvaro del Pino Gómez								
Aula	201-A	201-B	205	206	207	211	212	217-A	217-B
10:00 - 11:00	Geom. alg. II	Inf. est. frec. y apl.	Av. rec. en t. control	Rec. adv. local and nonlocal PDEs	Dif. geo., math. phys. and control th.	MatSI	Sistemas dinámicos y ED	Análisis armónico y t. operadores	Análisis funcional
11:00 - 11:30	Café - Hall 2								
Aula	201-A	201-B	205	206	207	211	212	217-A	217-B
11:30 - 13:30	Geom. alg. II	Topología	As. comp. y apl. de las EDP hiperbólicas	Rec. adv. local and nonlocal PDEs	Dif. geo., math. phys. and control th.	MatSI	Sistemas dinámicos y ED	Análisis armónico y t. operadores	Análisis funcional
13:30 - 15:30	Comida - Cafetería Dani y José								
15:30 - 16:30	Charla plenaria: Salón de Actos *Un paseo por el bosque: árboles y sus simetrías*, Alejandra Garrido								
16:30 - 17:00	Café - Hall 2								
Aula	201-A	201-B	205	206	207	211	212	217-A	217-B
17:00 - 19:00	Geom. alg. II	Topología	T. grupos	Rec. adv. local and nonlocal PDEs	Dif. geo., math. phys. and control th.	MatSI	T. grafos extremal. y aleat.	Análisis armónico y t. operadores	Análisis funcional
19:00 - 20:00	Charla - Salón de Actos *Bayes y el dardo perdido*, Anabel Forte								

VIERNES, 10 de febrero									
9:00 - 10:00	Charla plenaria: Salón de Actos *The fractional obstacle problem*, Xabier Fernández Real								
Aula	201-A	201-B	205	206	207	211	212	217-A	217-B
10:00 - 11:00		Topología	T. grupos	As. comp. y apl. de las EDP hiperbólicas	Inf. est. frec. y apl.	MatSI	Sistemas dinámicos y ED	Análisis armónico y t. operadores	Av. rec. en t. control
11:00 - 11:30	Café - Hall 2								
Aula	201-A	201-B	205	206	207	211	212	217-A	217-B
11:30 - 13:00		Topología	T. grupos			MatSI	Sistemas dinámicos y ED	Análisis armónico y t. operadores	Av. rec. en t. control
13:00 - 13:30	Acto de clausura - Salón de Actos								
13:30 - 15:30	Comida - Cafetería Dani y José								

Inauguración del congreso

El acto de inauguración del VI Congreso de Jóvenes Investigadores de la RSME tendrá lugar en el Salón de Actos de la Escuela de Ingenierías Industrial, Informática y Aeroespacial el lunes 6 de febrero de 2023, a las 9:00. La mesa inaugural estará compuesta por:

Juan Francisco García Marín, rector de la Universidad de León.

José Antonio Díez Díaz, alcalde de León.

Joaquín Barreiro García, director de la Escuela de Ingenierías Industrial, Informática y Aeroespacial.

Eva Gallardo, presidenta de la Real Sociedad Matemática Española.

Adriana Suárez Corona, presidenta del comité organizador.

Visita guiada

Los congresistas podrán disfrutar de una visita guiada a pie por la ciudad de León en la tarde del miércoles, 8 de febrero. Se organizarán dos turnos, uno a las 16:00 y otro a las 17:30. A lo largo de su recorrido contemplarán edificios tan emblemáticos como la catedral, el palacio de los Guzmanes, la casa Botines o la Real Colegiata de San Isidoro.

Cena de gala

La cena de gala del congreso se celebrará el miércoles, 8 de febrero a las 21:00, en el hotel Barceló León Conde Luna.

Charla de divulgación

Tendremos el placer de escuchar la charla de Anabel Forte, profesora Titular en la Universitat de València y divulgadora en su blog anabelforte.com y en twitter (@aAnaBayes), colaboradora del programa Raíz de 5 de Radio 5, dirigido por Santi Garcia Cremades. Esta charla está programada el jueves 9 a las 19:00 y estará abierta a toda la sociedad leonesa. La charla se titula Bayes y el dado perdido.

Acto de clausura

El acto de clausura del congreso tendrá lugar el viernes 10 de febrero, a las 13:00, en el Salón de Actos de la Escuela de Ingenierías Industrial, Informática y Aeroespacial. La mesa de clausura estará compuesta por:

Nicanor Sen, vicepresidente primero de la Diputación de León.

Javier Gómez, director del Departamento de Matemáticas de la Universidad de León.

Adriana Suárez Corona, presidenta del comité organizador.

Resúmenes - Conferencias Plenarias

En el congreso se impartirán siete charlas plenarias:

- *Maximal averages along variable manifolds*, por David Beltrán (Universitat de València)
- *¿Cómo distribuir puntos en la esfera?*, por Ujué Etayo (Universidad de Cantabria)
- *The fractional obstacle problem*, por Xavier Fernández-Real (École Polytechnique Fédérale de Lausanne)
- *Un paseo por el bosque: árboles y sus simetrías*, por Alejandra Garrido (Universidad Autónoma de Madrid – ICMAT)
- *Mathematical Optimization and Data Science: a back and forth relationship*, por Vanesa Guerrero (Universidad Carlos III de Madrid)
- *Relaciones diferenciales no-locales*, por Álvaro del Pino (Universiteit Utrecht)
- *Fundamentos Matemáticos del Aprendizaje Semi-Supervisado*, por Daniel Sanz-Alonso (University of Chicago)

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Maximal averages along variable manifolds

David Beltran*

Maximal functions are of fundamental importance in Harmonic Analysis due to its close connection with almost everywhere convergence questions. A central example is the Hardy–Littlewood maximal function which given a non-negative locally integrable function f and any point $x \in \mathbb{R}^d$ outputs the largest average of f over all balls centered at x. The Lebesgue space mapping properties of this operator imply then the almost everywhere convergence of such averages to $f(x)$ as the radius of the balls shrink to 0 for $f \in L^p(\mathbb{R}^d)$ if $1 \leq p \leq \infty$.

In 1976, Stein [1] introduced the spherical maximal function, in which the averages are taken over spheres as opposed to balls, and established that for $d \geq 3$ this is bounded on $L^p(\mathbb{R}^d)$ if and only if $p > \frac{d}{d-1}$; moreover, a.e. convergence results related to solutions of the wave equation followed. The 2-dimensional case, which corresponds to the most singular situation, was established 10 years later by Bourgain [2]. Stein and Bourgain's seminal work initiated the study of maximal functions associated to other types of singular averages. In these types of questions, the curvature of the underlying manifolds plays a key role.

In this talk we present two extensions of Bourgain's result, in two different directions. The first one is a 3-dimensional counterpart, in which the averages are taken along dilates of curves in $\mathbb{R}^3$ with non-vanishing curvature and torsion, such as the helix. We prove that the corresponding maximal function is bounded on $L^p(\mathbb{R}^3)$ if and only if $p > 3$. The second one is 2-dimensional, and it is associated to a family of variable singular planar curves that originate from considering the Heisenberg group analogue of Bourgain's circular maximal function (which is 3-dimensional) and restricting it to Heisenberg radial functions. We prove that such a maximal function is bounded on $L^p(\mathbb{H}^1)$ on the class of Heisenberg radial functions if and only if $p > 2$.

If time permits, we will also discuss variation norm analogues and regularity properties of such maximal functions.

The talk will be mostly based on joint works with Shaoming Guo, Jonathan Hickman and Andreas Seeger [3, 4].

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¿Cómo distribuir puntos en la esfera?

Ujué Etayo*

Las preguntas que rodean el tema de la distribución de puntos en la esfera de dimensión 2 son sencillas: ¿Cómo colocar 7 puntos en una esfera de forma que se maximice la mínima distancia entre dos pares de puntos cualesquiera?, ¿Qué configuraciones de electrones (pensemos en puntos) minimizan el potencial eléctrico en una esfera?, esas mismas configuraciones, ¿también minimizan el potencial logarítmico? Preguntas sencillas con una característica común: todas ellas permanecen abiertas.

A lo largo de esta charla nos introduciremos en el apasionante mundo de la distribución de puntos en la esfera 2-dimensional, estudiando las soluciones parciales a distintos problemas ahí planteados. Además, mostraremos como distintos problemas de áreas variadas de las matemáticas han encontrado solución a través de una buena distribución de puntos en la esfera.

Agradecimientos Nada de lo aquí expuesto sería posible sin el trabajo de muchos matemáticos antes que yo. Los que me han hecho posible entender tantos problemas son Carlos Beltrán, Jordi Marzo, Joaquim Ortega-Cerdà y Peter Grabner, entre otros.

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The fractional obstacle problem

Xavier Fernández-Real*

In this talk we present an introduction to integro-differential operators and the obstacle problem, and how they interact with each other in what is known as the *fractional obstacle problem*.

In $\mathbb{R}^n$, the fractional obstacle problem with a given obstacle $\varphi \in C_c^\infty(\mathbb{R}^n)$ can be written as

$$\text{mín}\{(-\Delta)^s u, u - \varphi\} = 0 \quad \text{in} \quad \mathbb{R}^n.$$

The set $\{u = \varphi\}$ is called the *contact set*, and its boundary $\partial\{u = \varphi\}$ is called the *free boundary*, which are unknowns of the problem. The free boundary can be divided into two sets: regular points (where it is regular) and degenerate points. In general, degenerate points can exist in any dimension.

We will finish the talk by presenting joint results with D. Torres-Latorre on the generic regularity of the free boundary. In particular, in dimension $n = 3$ we show that the free boundary of almost every solution is formed only of regular points.

*EPFL

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Un paseo por el bosque: árboles y sus simetrías

A. Garrido*

Un árbol es un grafo conexo carente de ciclos. Los árboles, sobre todo los finitos, aparecen en muchos ámbitos de matemáticas e informática. Los árboles infinitos también son importantes en geometría, topología, álgebra y teoría de números; donde aparecen como ejemplos de espacios métricos hiperbólicos, espacios recubridores de grafos, espacios simétricos de grupos de Lie (o más bien sus análogos sobre cuerpos p-ádicos donde p es un número primo), por mencionar algunos ejemplos.

Los grupos de simetrías de estos árboles son importantes no sólo por la información que contienen relacionada con los contextos en los que aparecen los árboles, si no también desde el punto de vista de la teoría de grupos.

Hablaré de algunos ejemplos de grupos de simetrías de árboles infinitos y de su relevancia en la teoría de grupos actual.

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Mathematical Optimization and Data Science: a back and forth relationship

Vanesa Guerrero [*]

In an era in which the decision-making is often based on the analysis of complex and evolving data, it is crucial to have methodologies and tools which provide helpful support to the decider and that ease the interpretation of the models and results driving the process. Many of the problems arising in this context can be addressed combining the theoretical frameworks of mathematical optimization and data science, thus fostering knowledge exchange in both directions. On the one hand, addressing traditional data analysis modeling approaches from the perspective of mathematical optimization allows to incorporate desirable properties to the analysis. On the other hand, statistical modeling can help mathematical optimization to develop formulations which are more tractable in practice.

In this talk three bidirectional collaborations between mathematical optimization and data science will be discussed. First, the problem of enhancing interpretability in some dimensionality reduction techniques through sparsity [1] and using exogenous information [3] is addressed by means of Mixed Integer Nonlinear Programming (MNILP) approaches. Second, the usefulness of information visualization to improve interpretability of the unknown phenomena described by row data will be discussed. In particular, we focus on the use of global optimization to develop generalizations of the standard multidimensional scaling for which difference of convex functions and nonconvex quadratic binary optimization techniques are combined as solution approaches [2]. Lastly, we address the problem of reformulating general MINLPs using regression splines. The choice of these functional forms allows us to approximate general non-convex MINLPs by a more tractable subclass of problems which can be efficiently solved by customized algorithms [4].

Acknowledgments The researcher is supported by projects PID2019-104901RB-I00, PID2019-110886RB-I00 and IJC2020-045220-I (MCIN/AEI/10.13039/501100011033 and the last also by European Union "NextGenerationEU/PRTR" funds), FQM-329, P18-FR-2369 and US-13811 (Junta de Andalucía) and PGMO program (Fondation Mathématique Jaques Hadamard). This support is gratefully acknowledged.

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Relaciones diferenciales no-locales

A. del Pino Gómez*

Muchas de las estructuras que estudiamos en Geometría Diferencial tienen una naturaleza que es local. Por ejemplo, comprobar que un mapa es una inmersión es algo que hacemos punto a punto, mirando que la diferencial tenga rango máximo.

Por otro lado, los encajes son los ejemplos típicos de estructuras geométricas no-locales. Comprobar que un mapa es un encaje requiere comparar pares de puntos en el dominio y ver que su imagen es diferente.

Existe una literatura amplia estudiando encajes, combinando técnicas de Topología Diferencial y Teoría de Homotopía (algunos nombres relevantes: Haefliger, Goodwillie, Klein, Weiss). Sin embargo, no hay una teoría general que trate estructuras geométricas no-locales arbitrarias.

El objetivo de la charla (que pretende ser elemental) es introducir este círculo de ideas, describir algunos resultados recientes obtenidos con A. Gootjes-Dreesbach, y especular un poco sobre qué más puede hacerse en esta dirección.

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Fundamentos Matemáticos del Aprendizaje Semi-Supervisado

A. Daniel Sanz-Alonso*

En esta charla plantearé algunos problemas matemáticos y estadísticos que subyacen la formulación Bayesiana del aprendizaje semi-supervisado [2]. El objetivo de esta modalidad de aprendizaje es asignar etiquetas a un gran número de datos usando pocos ejemplos. En la formulación Bayesiana, la medida posterior sobre la función de etiquetado combina la verosimilitud de los ejemplos con una medida *a priori* que actúa como regularización. Describiré cómo definir la medida *a priori* utilizando procesos Gaussianos en grafos [5], las propiedades de concentración de la medida posterior [6], y técnicas de muestreo cuya tasa de convergencia no deteriora con el número de datos a etiquetar [1]. Finalmente, mencionaré ejemplos de otras aplicaciones donde ideas parecidas pueden ser útiles, como en la solución de problemas inversos [3, 4] y optimización [7] en variedades.

Agradecimientos La investigación resumida en esta presentación ha contado con el apoyo de las siguientes instituciones: Department of Energy, National Geospatial Intelligence Agency, National Science Foundation, Fundación BBVA.

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De profesión: matemático haciendo I+D+i

Mesa redonda que abarca la visión desde diferentes puestos profesionales que pueden ocupar actualmente los matemáticos, en la que se discutirán las oportunidades que ofrece cada alternativa, el tipo de tareas que se realizan, su profundidad matemática y el futuro de la carrera profesional de los matemáticos.
Para esta mesa redonda contaremos con matemáticos que ejercen su profesión en distintas empresas. Será moderada por Marcos Gómez, Subdirector de INCIBE.

Moderador: Marcos Gómez	*Subdirector de INCIBE*
Alfonso D. Bláquez	*Freelance. The Data Warlock*
Covadonga Fernández	*Head of Credit Risk Model Validation. Banco Santander*
Almudena Sastre Noriega	*Team leader en el equipo SAP. NTT Data Valladolid*
Laura Toribio	*Postdoctoral Researcher. CIEMAT*

Matemátic@s en educación

Mesa redonda que abordará cuestiones relativas a la profesión de Profesor de Matemáticas tanto en el ámbito de la Educación Universitaria, como en el de la Educación Secundaria. Comentaremos cómo acceder a cada una de ellas visualizando ciertos escollos; desde la realización del máster de Formación de Profesorado, al complicado sistema de evaluaciones de la ANECA y las distintas figuras de profesor (haciendo alusión a la nueva ley de Universidades LOSU). Abordaremos los problemas que podemos encontrarnos en el ejercicio, como la motivación de nuestros alumnos. Así mismo, trataremos las recompensas que ofrece la profesión, bajo la visión de profesores en ejercicio. Sin dejar de intentar dar respuesta a la falta de profesorado de matemáticas en la Educación Secundaria.

Moderadora: Itziar García-Honrado	*Profesora Titular de la Universidad de Oviedo*
Sara Álvarez	*Profesora de Secundaria y Profesora Asociada de la Universidad de Oviedo*
Daniel Cao	*Profesor Ayudante Doctor de la Universidad de Santiago de Compostela*
Pablo Martín	*Profesor de Secundaria, Profesor Asociado en la Universidad de León y presidente de la asociación Castellano-Leonesa de educación matemática "Miguel de Guzmán"*
Luis José Rodríguez Muñiz	*Catedrático de la Universidad de Oviedo y Vicepresidente segundo de la RSME*

Resúmenes - Sesiones paralelas

Las contribuciones del congreso están organizadas en las siguientes sesiones paralelas:

1. Topología
2. Geometría simpléctica y dinámica Hamiltoniana
3. Geometría convexa
4. Geometría diferencial
5. Differential geometry, mathematical physics and control theory
6. Teoría de grupos
7. Álgebras no asociativas y aplicaciones
8. Teoría de singularidades
9. Geometría algebraica I: Fundamentos
10. Geometría algebraica II: Aplicaciones
11. Teoría de grafos extremales y aleatorios
12. Investigación operativa
13. Matemáticas en la sociedad de la información
14. Inferencia estadística frecuentista y aplicaciones
15. Avances y aplicaciones de la estadística
16. Teoría de la probabilidad
17. Análisis funcional
18. Análisis armónico y teoría de operadores
19. Sistemas dinámicos y ecuaciones diferenciales
20. Aspectos computacionales y aplicaciones de las ecuaciones en derivadas parciales hiperbólicas
21. Mathematical challenges in fluid dynamics and kinetic theory
22. Análisis Matemático Aplicado y Ecuaciones Diferenciales
23. Recent advances in local and nonlocal PDEs
24. Avances Recientes en Teoría de Control

Topología

Organizadores: Mario Fuertes y Sergio García

Jueves 9 de Febrero
Moderador: Mario Fuentes

11:30 *Derivadores y homotopía: el cálculo de límites*, Beatriz Álvarez

12:00 *Cohomología de haces para CL-shellable posets*, Guille Carrión

12:30 *Estabilidad de Sucesiones Espectrales de Mayer–Vietoris*, Álvaro Torras

13:00 *Descomposiciones simpliciales y campos de vectores discretos*, Juan Delgado

17:00 *Un ejemplo "elemental" de cuantización por deformación con Álgebras de Factorización*, Víctor Carmona

17:30 *Computing signatures of nearby fibres of parametrized complex surfaces*, Roberto Giménez

18:00 *Grupo de cobordismos Lagrangianos*, Álvaro Muñiz

18:30 *Una introducción (muy introductoria) a la (co)homología de intersección*, Alba Sedón

Viernes 10 de Febrero
Moderador: *Sergio García*

10:00 *Apollonian packings and knots*, Iván Rasskin

10:30 *Homología de Khovanov y enlaces ligados*, O'Bryan Cárdenas

11:30 *The cohomological index of free Z/p-actions*, Rafael Gomes

12:00 *Distancia homotópica entre aplicaciones*, David Mosquera

12:30 *Homotopía racional parametrizada estable con modelos de Lie*, Alejandro Saiz

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Derivadores y homotopía: el cálculo de límites

B.Álvarez Díaz*

La idea de derivador aparece por primera vez de la mano de Grothendieck [1] y Heller [2] de forma independiente, siendo posteriormente desarrollada por autores como Franke [3], Keller [4] y Maltsiniotis [5].

Desde su nacimiento, los derivadores han estado ligados a la teoría de homotopía. Nuestra intención será utilizar este concepto como herramienta para establecer un cálculo de límites y colímites homotópicos en ciertas categorías, aprovechando su definición axiomática como 2-funtores.

En primer lugar, partiendo de una categoría de Grothendieck A, nos centraremos en la categoría derivada de la categoría de complejos $\mathsf{D}(\mathsf{A})$. Keller demuestra en [6] que la categoría derivada tiene estructura de derivador para categorías acotadas con dominio en categorías esencialmente finitas y Cisinski prueba en [7] que cualquier categoría modelo completa y cocompleta define un derivador.

El objetivo en este caso será dar una descripción explícita del derivador asociado a $\mathsf{D}(\mathsf{A})$ y de su utilidad para el cálculo de límites y colímites homotópicos en dicha categoría [8].

Una vez obtenidos estos límites y colímites homotópicos en la categoría derivada, podríamos pensar que el prodecimiento es extrapolable al caso de la categoría homotópica de complejos, $\mathsf{K}(\mathsf{A})$. Veremos que en este caso existen elementos que entran en conflicto con la definición del derivador, lo cual requiere un refinamiento de esta noción.

Agradecimientos Este trabajo ha sido parcialmente financiado por el proyecto MTM2017- 89830-P del MINECO de España, por el proyecto ED431C 2019/10 de la Xunta de Galicia y por el proyecto de investigación PID2020-115155GB-I00 del MCINN, todos con Fondos FEDER Europeos. Además la autora ha sido financiada a través del contrato FPU18/01203.

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Homología de Khovanov y enlaces ligados

O'Bryan Cárdenas-Andaur*

En el año 2000, Mikhail Khovanov introduce el primer invariante homológico de nudos y enlaces, hoy conocido como Homología de Khovanov [1], el cual se obtiene como una categorificación del polinomio de Jones y resulta ser un invariante más fino que este (detecta el nudo trivial).

Aicardi y Juyumaya, por otro lado, presentan en el año 2016 los enlaces ligados o tied links [2], los cuales constituyen una generalización no trivial de los enlaces clásicos. Así también definen invariantes polinomiales para estos nuevos objetos, en particular, en el 2018 presentan el invariante $\langle\langle\cdot\rangle\rangle$ que generaliza el corchete de Kauffman y permite construir una generalización del polinomio de Jones, por el método de Kauffman, para enlaces ligados [3].

El objetivo de esta charla es, en primer lugar, introducir el polinomio de Jones y la homología de Khovanov para enlaces clásicos. En segundo lugar presentar los enlaces ligados y el invariante $\langle\langle\cdot\rangle\rangle$. Finalmente, mostrar ejemplos para ilustrar los pasos y complejidades para construir una Homología de Khovanov para enlaces ligados utilizando el invariante $\langle\langle\cdot\rangle\rangle$.

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Un ejemplo “elemental” de cuantización por deformación con Álgebras de Factorización

Victor Carmona*

Un famoso teorema de Kontsevich demuestra que toda variedad de Poisson admite una cuantización por deformación (deformation quantization en inglés de la calle). Este resultado respondió a una pregunta fundamental en Física Matemática y su elegante prueba abrió todo un campo de investigación. En pocas palabras, Kontsevich demostró que las álgebras de Lie graduadas que controlan el problema de deformación de estructuras de Poisson y el análogo para álgebras asociativas, cierta homología de Hochschild, son equivalentes. Pero esta charla no va de esto, esta charla es mucho más elemental. Nosotros hablaremos de un caso particular de este teorema, el caso de estructuras de Poisson constantes, en el que la cuantización por deformación se puede construir de manera completamente geométrica usando álgebras de factorización. Para que la charla tenga algún sentido, repasaremos qué es eso de cuantización por deformación, qué es un álgebra de factorización y como este ejemplo que nos proponemos presentar es posible gracias a la "trivialidad de la renormalización"para Teorías de Campos Topológicas. Esta charla se basa en un trabajo conjunto con el Profesor Damien Calaque (IMAG).

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VI Congreso de Jóvenes Investigadores

Real Sociedad Matemática Española

León, Febrero de 2023

Cohomología de haces para CL-shellable posets

Guille Carrión Santiago*

Un haz en un espacio topológico finito $\mathcal{P}$ es un funtor (contravariante) $F\colon \mathcal{P}^{op} \to R-$ Mod, donde $\mathcal{P}$ es visto como una categoría parcialmente ordenada (poset en adelante) y R un anillo conmutativo y con unidad. Los grupos de cohomología de $\mathcal{P}$ con coeficientes en el haz F se definen como el funtor derivado del funtor límite evaluado en F:

$$H^*(\mathcal{P}; F) \colon = R^* \lim_{\mathcal{P}} (-)(F).$$

En esta charla calcularemos la cohomología de haces en poset con la propiedad de ser CL-shellable. Decimos que un poset es CL-shellable si existe una graduación λ en su relación de cobertura de forma que para cada intervalo $[x, y]$, existe una única cadena λ-creciente maximal que además es lexicográficamente minimal.

Para calcular los funtores derivados del funtor límite presentaremos una estructura de modelos en la categoría de funtores $\mathrm{Fun}(\mathcal{P}, \mathrm{Ch}(R))$, gracias a la cual calculamos la cohomología de un haz F utilizando un reemplazo fibrante:

$$H^*(\mathcal{P}; F) = H^*(\lim \mathbf{R}F),$$

donde $\mathbf{R}F\colon \mathcal{P}^{op} \to \mathrm{Ch}(R)$ es un funtor fibrante que es *homótopo* a F. Por último, utilizando la estructura combinatoria CL-shellable del poset describiremos propiedades del haz que impliquen que, en ciertos grados, la cohomología del haz se anule.

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Descomposiciones simpliciales y campos de vectores discretos

J. A. Delgado Tejada*

La Homología Efectiva [4] es una teoría creada para facilitar el cálculo explícito de grupos de homología de un espacio topológico. La idea central es establecer una secuencia de reducciones, denotadas $C_* \Rightarrow D_*$, hasta alcanzar un complejo de tipo finito, en el que el cálculo de la homología puede ser realizado por operaciones matriciales conocidas.

Los campos de vectores discretos, introducidos por Forman en [2], proporcionan reducciones. Por lo tanto, encontrar un campo de vectores discretos en el complejo de cadenas de un espacio topológico X es un primer paso para calcular su homología efectiva.

El objetivo de la charla será presentar estos conceptos y tratar de abordar el problema de calcular campos de vectores discretos en algunos ejemplos concretos. Inspirados por un campo de vectores ya conocido [3] que induce la reducción de Eilenberg-Zilber:

$$C_*^N(X \times Y) \Rightarrow C_*^N(X) \otimes C_*^N(Y),$$

describiremos otro campo de vectores que induce una reducción del espacio clasificante sobre la construcción Bar

$$C_*^N(\overline{W}(G)) \Rightarrow \mathrm{Bar}^{(\mathbb{Z},\mathbb{Z})}(C_*^N(G)).$$

Además, presentaremos un trabajo en curso para intentar replicar una construcción del mismo estilo en el caso, más complicado, de los espacios de lazos:

$$C_*^N(\Omega X) \Rightarrow \mathrm{Cobar}^{(C_*^N(X))}(\mathbb{Z},\mathbb{Z}).$$

Finalmente, comentaremos cómo es posible manipular computacinoalmente estos campos de vectores a través de su implementación en `Kenzo` [1].

Agradecimientos Proyecto nacional: Ayuda PID2020-116641GB-I00 financiada por MCIN/AEI/ 10.13039/501100011033

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VI Congreso de Jóvenes Investigadores

Real Sociedad Matemática Española

León, Febrero de 2023

Computing signatures of nearby fibres of parametrized complex surfaces

Roberto Giménez Conejero*

Given a (good) map germ $f : (\mathbb{C}^2, 0) \to (\mathbb{C}^3, 0)$, the image is a complex surface in $\mathbb{C}^3$. This has an equation $H(x, y, z) = 0$. Moreover, the surface F given by $H(x, y, z) = \epsilon$, for $0 < |\epsilon|$ small, is smooth and does not depend on ϵ. Some details are missing.

In this joint work (with Pintér Gergő) in progress, we give a way to compute the signature of F, using immersion theory, that depends only on the topological type of f.

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The cohomological index of free $\mathbb{Z}/p$-actions

Rafael Gomes*

Let p be an odd prime, $\tilde{X}$ a space with a $\mathbb{Z}/p$-action and consider its orbit space X and the classifying map $f : X \to B\mathbb{Z}/p$. The cohomological index, denoted $ind(X)$, of this action is the smallest integer k for which the map $H^k(B\mathbb{Z}/p;\mathbb{Z}/p) \to H^k(X;\mathbb{Z}/p)$ induced in cohomology by f is trivial. In this talk, we will explore the properties of this invariant of free $\mathbb{Z}/p$-actions and particularly how additivity with respect to the (topological) join operation fails. As observed by Hara and Kishimoto in their study of the height of a covering, $ind(X) - 1$ is a lower bound for the L–S category of the classifying map. This is joint work with Gustavo Granja.

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Distancia homotópica entre aplicaciones

David Mosquera Lois[*]

Sean $f, g\colon X \to Y$ dos aplicaciones continuas entre espacios topológicos. En esta comunicación presentaremos un invariante homotópico, llamado distancia homotópica entre las aplicaciones f y g, y denotado $\mathrm{D}(f, g)$. Además, veremos algunas propiedades y resultados notables sobre la distancia homotópica y su relación con otros invariantes clásicos, por ejemplo la categoría de Lusternik–Schnirelmann y la complejidad topológica.

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Grupo de cobordismos Lagrangianos

Álvaro Muñiz*

Los cobordismos Lagrangianos son un objeto fundamental en topología simpléctica. Introducidos por V. Arnold en los años 80 como un intento de clasificación de subvariedades Lagrangianas, más recientemente el trabajo de P. Biran y O. Cornea en 2013 muestra que estos objetos geométricos detectan la estructura triangulada de la categoría de Fukaya (invariante simpléctico más potente hasta la fecha). En esta charla consideraremos el grupo de cobordismos Lagrangianos, un grupo abeliano generado por las subvariedades Lagrangianas y con relaciones dadas por cobordismos Lagrangianos. La charla comenzará con un breve repaso de topología simpléctica, la definición de dicho grupo y ciertas propiedades; finalmente, presentaremos un cálculo explícito para el 2-toro y, si el tiempo lo permite, un resultado estructural sobre la infinito-dimensionalidad del grupo. (En la comparación entre el grupo de cobordismos Lagrangianos y los grupos de Chow sugerida por la conjetura de simetría especular homológica, estos son los resultados "espejo" del grupo de Chow de una curva elíptica y el teorema de Mumford.)

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Apollonian packings and knots

I. Rasskin *, J. L. Ramírez Alfonsín †

Apollonian packings and its generalizations have been the source of many applications in different fields of mathematics [2]. In this paper, we use them to present new geometric representations of knots and links which turn out to be useful in several fronts.

We exploit such representations to improve the general upper bound of the ball number given in [1] for an infinite family of algebraic links. We also show that every rational tangle $t_{p/q}$ can be regularly projected on the boundary of an Apollonian packing in $\widehat{\mathbb{C}}$, in such a way that the projection intersects the real line at the rational numbers appearing in the path joining $\frac{0}{1}$ and $\frac{p}{q}$ in the Stern-Brocot tree. The latter gives a new visualization of the arithmetic structure behind rational tangles.

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León, Febrero de 2023

Homotopía racional parametrizada estable con modelos de Lie

Alejandro Saiz*

La homotopía racional estudia el comportamiento “sin torsión” del tipo de homotopía de los espacios topológicos. Con la reciente revisión del enfoque de Quillen de esta teoría en la que caben espacios no necesariamente simplemente conexos, pretendemos dar un nuevo marco para el estudio de la teoría de homotopía estable parametrizada desde un punto de vista racional.

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León, Febrero de 2023

Una introducción (muy introductoria) a la (co)homología de intersección

Alba Sendón Blanco*

La dualidad de Poincaré es uno de los teoremas matemáticos más célebres: si M es una n-variedad cerrada, orientada y conexa, el cap-producto con la clase fundamental induce un isomorfismo $H^i(M) \cong H_{n-i}(M)$ para todo i. Esta propiedad falla para espacios más generales.

La homología de intersección de M. Goresky y R. MacPherson [4], basada en una noción de perversidad, restituye la dualidad de Poincaré para algunos espacios con singularidades como las pseudovariedades.

Para obtener un isomorfismo de Poincaré en este escenario, necesitamos una teoría de cohomología de intersección. El primer candidato proviene del dual lineal [3], y otra opción más elaborada se basa en un blow-up simplicial [1].

En la charla, explicaremos la homología de intersección y las dos teorías de cohomología, viendo algunos resultados sobre dualidad [2] conseguidos hasta el momento con cada una de las alternativas.

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Estabilidad de Sucesiones Espectrales de Mayer–Vietoris

A. Torras Casas,* U. Pennig*

La homología persistente puede requerir una alta cantidad de recursos computacionales. Una cuestión que se ha explorado en la última década consiste en conseguir distribuir los cálculos mediante la sucesión espectral de Mayer–Vietoris. Sea X un complejo regular filtrado y sea un recubrimiento de X compuesto de subcomplejos. Denotamos por N el *nervio* de y escribimos ${}_\sigma = \bigcap_{U\in\sigma} U$ para todo $\sigma \in N$. En [1] se propuso un método para dividir los cálculos de homología persistente mediante el *complejo explosivo de Mayer-Vietoris* Δ_X. Este método se basa en la equivalencia homotópica $X \simeq \Delta_X$ y en una filtración F_* de Δ_X tal que existen isomorfismos, para todo $1 \leq i \leq n$: $F_i\Delta_X \big/ F_{i-1}\Delta_X \simeq \coprod_{\sigma\in N^i}{}_\sigma$, donde N_i denota el subconjunto de i-simplices de N. Esta misma idea se estudió más tarde [2, 3] mediante el uso de sucesiones espectrales [4]. En particular, consideramos la sucesión *espectral de Mayer–Vietoris* $E^1_{p,q}(X,) = \bigoplus_{\sigma\in N^n}{}_q({}_\sigma) \Rightarrow {}_{p+q}(X)$; donde ${}_q$ denota la homología persistente en dimensión q. Esto permite obtener ${}_q(X)$ partir de un cálculo paralelo de homologías persistentes [5]. En particular, el resultado final contiene información sobre la *localización* [6] de ${}_*(X)$ con respecto de . En este contexto, nos planteamos las siguientes preguntas: ¿Cuál es la estabilidad de $E^1_{p,q}(X,)$? ¿Qué ocurre si cambiamos el recubrimiento ? ¿Podríamos generalizar este resultado?

Supongamos que tenemos dos complejos filtrados X e Y. Consideramos dos recubrimientos y para X e Y respectivamente. Tenemos el siguiente resultado de estabilidad. Supongamos que existe un isomorfismo $\phi : N_{\to}N$ tal que existen -equivalencias acíclicas ${}_\sigma \rightrightarrows_{\phi(\sigma)}$ y que además satisfacen una condición de compatibilidad; ver [7]. Entonces, existe una -equivalencia acíclica $\Delta_X \rightrightarrows \Delta_Y$ tal que preserva la filtración F_*. Como consecuencia de este resultado obtenemos que ${}_n(F_i\Delta_X)$ y ${}_n(F_i\Delta_Y)$ están -intercalados para todo $i \geq 0$. Más aún, en [7] se obtiene una -intercalación entre las primeras páginas de $E^1_{*,*}(X,)$ y $E^1_{*,*}(Y,)$.

Consideramos un complejo filtrado X junto con dos recubrimientos y arbitrarios. En este caso es posible adaptar resultados de J. P. Serre [8] para obtener $R(,)$-intercalaciones entre las segundas páginas $E^2_{*,*}(X,)$ y $E^2_{*,*}(Y,)$. El parámetro $R(,) > 0$ depende de un cálculo que se puede realizar en paralelo y que compara los recubrimientos y a nivel local.

Si el tiempo lo permite, explicaré brevemente que estos resultados pueden adaptarse al caso de diagramas de espacios. Un ejemplo que motiva esta generalización es el *diagrama de juntura* [7], que permite distribuir los cálculos de persistencia homológica eficientemente para el caso de complejos simpliciales filtrados. En este caso general, consideramos la sucesión espectral asociada a la realización geométrica que tiene bastantes relaciones con la sucesión espectral del colímite homotópico.

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Geometría Simpléctica y Dinámica Hamiltoniana

Organizadores: Robert Cardona, Mar Giralt y Cédric Oms

Lunes 6 de Febrero
Moderador: *Robert Cardona*

12:30 *Semi-analytical computation of center-stable and center-unstable manifolds in the geostationary belt*, Miquel Barcelona.

13:00 *La conjetura de Arnold para variedades b^m-simplécticas*, Joaquim Brugués.

18:15 *Integrabilidad en sistemas hamiltonianos de contacto*, Asier López-Górdon.

Martes 7 de Febrero
Moderadora: *Mar Giralt*

10:00 *Positive measure of effective quasi-periodic motion near a diophantine torus*, Gerard Farré

10:30 *Integración de foliaciones singulares usando caminos*, Alfonso Garmendia

11:30 *Parabolic ejection and collision orbits for the restricted planar circular three body problem*, José Lamas

12:00 *Holomorphic h–principles over Stein manifolds*, Guillermo Sánchez

12:30 *Periodic perturbation of a 3D conservative flow with 2D heteroclinic connection to saddle-foci*, Ainoa Murillo

13:00 *Introduction to Contact Topology*, Dahyana Farias

18:15 *An overview about bracket-generating distributions*, Javier Martínez-Aguinaga

Miércoles 8 de Febrero
Moderador: *Cédric Oms*

10:00 *Bohr-Sommerfeld quantization of b-symplectic toric manifolds*, Pau Mir

10:30 *Splitting of separatrices for rapidly forced pendulum with a perturbation without first harmonic*, Román Moreno

11:30 *Hamiltonian facets of classical gauge theories on E-manifolds*, Pablo Nicolás

12:00 *A degenerate Arnold Diffusion Mechanism in the Restricted 3 Body Problem*, Jaime Paradela

12:30 *A simple proof of Novikov's theorem*, Samuel Ranz

13:00 *Unique ergodicity of the horocyclic flow of surfaces without conjugate points*, Sergi Burniol

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León, Febrero de 2023

Semi-analytical computation of center-stable and center-unstable manifolds in the geostationary belt

M. Barcelona [*], A. Haro [†], J.M.. Mondelo[*]

This talk is devoted to present a methodology for the computation of center-stable and center-unstable manifolds of center$\times$center$\times$saddle fixed points of autonomous Hamiltonian systems. The methodology involves the computation of Taylor expansions of center-unstable and center-stable manifolds of fixed points through the parameterization method, using a new style that uncouples the center part from the hyperbolic one, thus making the fibered structure of the manifolds explicit. We also present how to uniformly mesh iso-energetic slices of the center manifold, using a novel strategy that avoids numerical integration of the reduced differential equations and makes an explicit 3D representation of these slices as deformed ellipsoids. The method is applied to the computation of center-stable and center-unstable manifolds of the unstable equilibrium points in the geostationary belt. In this talk, we use these results to analyse the invariant manifold structure for whole iso-energetic slices of the center manifold of these fixed points. Computations related to the hyperbolic part will also be considered.

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León, Febrero de 2023

La conjetura de Arnold para variedades b^m-simplécticas

Joaquim Brugués*

Las variedades b^m-simplécticas (comúnmente también llamadas $\log$-simplécticas) son un caso concreto de variedades de Poisson que son simplécticas fuera de una hipersuperficie (véase por ejemplo [4] o [3]) pero degeneradas en dicha hipersuperficie. Su estudio emergió naturalmente de la investigación de estructuras simplécticas en variedades con borde [8], pero también juegan un papel en el espacio de geodésicas del plano de Lorentz [6] y tienen aplicaciones como espacios de fase para determinadas regularizaciones del problema de los tres cuerpos restringido [2].

Pese a no ser propiamente estructuras simplécticas, las estructuras b^m-simplécticas exhiben una rigidez similar. Por ejemplo, puede aplicarse un teorema de Moser análogo al de la geometría simpléctica, lo cual lleva a un teorema de Darboux equivalente al del caso simpléctico [4]. También hay una cohomología análoga a De Rham [7] que depende de la cohomología de la variedad y de la hipersuperficie y que calcula la cohomología de Poisson de la variedad, además de dar algunas restricciones topológicas para el caso de variedades b^m-simplécticas compactas. Por lo tanto, uno puede conjeturar que los fenómenos estudiados en dinámica y topología simplécticas tienen paralelos en el contexto b^m-simpléctico.

En esta charla explicaremos los resultados de un proyecto conjunto con Eva Miranda y Cédric Oms en el cual buscamos reproducir la conjetura de Arnold para variedades b^m-simplécticas.

La conjetura de Arnold es una proposición ampliamente estudiada en geometría simpléctica que asevera que el número mínimo de órbitas periódicas de periodo 1 de un campo Hamiltoniano no degenerado en una variedad compacta es igual al número mínimo de puntos críticos de una función de Morse en dicha variedad. Dicha conjetura tiene a día de hoy una prueba completa a la cual han contribuido numerosos autores, notablemente Andreas Floer con la introducción de su teoría homológica (véase [9] o [1] para una cronología detallada de las contribuciones a esta demostración)

En nuestro proyecto probamos que la misma cota es aplicable al caso b^{2m}-simpléctico usando una técnica de desingularización [5], y que una cota aún más precisa puede hallarse para superficies b^m-simplécticas.

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Unique ergodicity of the horocyclic flow of surfaces without conjugate points

S. Burniol Clotet*

There are strong connections between the dynamics of the geodesic flow and the horocyclic flow defined on the unit tangent bundle of certain Riemannian surfaces. H. Furstenberg first proved that the horocyclic flow of a compact hyperbolic surface is uniquely ergodic [1], i.e. it admits a unique invariant probability measure. Then B. Marcus generalized the result to negative variable curvature [2].

I will explain why this result still holds for a compact surface without conjugate points, genus greater than 1 and continuous Green bundles. The result applies in particular to nonpositively curved nonflat surfaces and is also new for this class. In our setting, the lack of uniform hyperbolicity of the geodesic flow is the main difficulty and makes the result nontrivial.

The proof uses the construction of the measure of maximal entropy for the geodesic flow in a recent paper of V. Climenhaga, G. Knieper and K. War [3], and the semiconjugation of the geodesic flow with an expansive continuous flow with local product structure, established by K. Gelfert and R. Ruggiero [4]. We define a suitable parametrization of the horocycle flow on an auxiliary space and apply Marcus's ideas.

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VI Congreso de Jóvenes Investigadores

Real Sociedad Matemática Española

León, Febrero de 2023

Introduction to Contact Topology

Dahyana Farias Uncovich*

In this talk we are going to start giving basic concepts of contact topology through examples and some applications.

This area starts with Lie in 1872 introducing the idea of the contact transformations, to use as a tool to solve systems differential equations.

For this reason, it plays an important role in applied mathematics as mechanic, optical, thermodynamic and control theory.

For theory mathematics the contact topology has a recent origin. The topologies methods have begun to be important in the contact geometry for 1970 decade but the topologies global results remained isolated until 1980 decade.

On the order hand, we are going to answer the following questions:

- Is it possible to have more than one contact structure?
- What type of relation have between them?
- Is there contact submanifold?
- What type of relation have contact topology and symplectic topology?

At last, we are going to define one type of contact structure called singular contact Beltrami, and we will mention the goals that we want to reach out.

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Positive measure of effective quasi-periodic motion near a diophantine torus

G. Farré Puiggalí *

In his ICM lecture in 1998 Michael Herman conjectured that given a real analytic Hamiltonian system $H : \mathbb{T}^d \times \mathbb{R}^d \longrightarrow \mathbb{R}$ of the type

$$H(\theta, I) = \omega \cdot I + \mathcal{O}(|I|^2)$$

with $\omega \in \mathbb{R}^d$ a diophantine vector, the invariant diophantine quasi-periodic torus $\mathcal{T} = \mathbb{T}^d \times \{0\}$ is accumulated by a set of invariant quasi-periodic tori with positive measure (see [1]). In this talk I will present a weaker result by looking at this conjecture from the perspective of effective stability, where we focus on studying how much time the corresponding orbits to initial conditions near $\mathcal{T}$ are close to having a quasi-periodic behaviour, and so for how much time the solutions are "effectively quasi-periodic" around $\mathcal{T}$. It is well-known that for this type of systems the motion near $\mathcal{T}$ is effectively quasi-periodic for an exponentially long time on the inverse of the distance to $\mathcal{T}$ (see [2, 3]). I will present a result stating that the motion around $\mathcal{T}$ is effectively quasi-periodic for a double exponentially long time on the inverse of the distance to the torus, for a set of initial conditions which has Lebesgue density equal to one at $\mathcal{T}$. Furthermore, the complement of this set is exponentially small. This is a joint work with A. Bounemoura.

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Real Sociedad Matemática Española

León, Febrero de 2023

Integracion de foliaciones singulares usando caminos

A. Alfonso Garmendia [*], B. Joel Villatoro [**]

El término "integracion" es usado en muchos contextos. El más básico de estos es el referente a encontrar el área de bajo una curva. Debido al teorema fundamental del cálculo, este concepto básico de integración es equivalente a antiderivar, o a la resolución de una ecuación diferencial. En un contexto más general, integrar es resolver una ecuación diferencial.

En el contexto álgebras de Lie, integración es encontrar un grupo de Lie cuyo espacio tangente es sea isomorfo al álgebra de Lie inicial. Este concepto de integración está ligado al de resolver una ecuación diferencial. Las derivaciones son un álgebra de Lie y su solución está dada por un flujo, el cual es la acción de un grupo de Lie.

Las derivaciones en una variedad están dadas por campos vectoriales. Bajo ciertas condiciones una colección de campos vectoriales forman una foliación y un álgebra de Lie infinito dimensional o un algebroide de Lie (finito dimensional).

Una manera topología-algebraica-diferencial para integrar un álgebra de Lie de dimensión finita y un algebroide de Lie, es tomar una colección de caminos y hacer un cociente por cierta homotopía [2]. El resultado de la integración de un algebroide de Lie es un grupoide.

Las foliaciones singulares están cerca de ser un algebroide de Lie. Siguiendo la construcción en [2], en esta charla presentaré una la estructura difeológica del espacio de caminos a lo largo de una foliación singular. Estos caminos son usados para integrar la foliación singular dando como resultado un grupoide.

Por otro lado en [1] se presenta otra integración diferente usando cartas. Esta forma de integración da una estructura sobre el grupoide similar al de una variedad.

Este fue un trabajo en conjunto con Joel Villatoro para un artículo llamado "Integration of singular foliations via paths" [3].

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VI Congreso de Jóvenes Investigadores
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León, Febrero de 2023

Parabolic ejection & collision orbits for the restricted planar circular three body problem

A. Marcel Guàrdia Munárriz[*] B. José Lamas Rodríguez[†] C. Tere M-Seara [‡]

We consider the restricted planar circular three body problem (RPCTBP), which describes the motion of a massless body under the attraction of other two bodies, the primaries, which describe circular orbits around their common center of mass located at the origin.

In a suitable system of coordinates, this system is Hamiltonian with two degrees of freedom, whose conserved energy is usually called the Jacobi constant. In such system, we are interested in solutions of the RPCTBP called *ejection-collision orbits*, i.e., solutions that depart from the big primary at some time t_0 and collide with it at some instant t_1. In particular, we will study the case when such orbits go arbitrarily far away for small values of the mass ratio.

To obtain them, we show that, for small values of the mass ratio and the Jacobi constant, there exist transversal intersections between the stable (unstable) manifold of *infinity* and the unstable (stable) manifold of collision.

Close to such transversal intersections, we prove the existence of a sequence of ejection-collision orbits that travel arbitrarily far away. Moreover, using a similar argument, we prove the existence of a sequence of forward and backward periodic parabolic orbits that travel close to collision too. Finally, we also prove the existence of periodic orbits that travel close to collision and arbitrarily far away.

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Real Sociedad Matemática Española

León, Febrero de 2023

Integrabilidad en sistemas hamiltonianos de contacto

Asier López Gordón,[*] Leonardo Colombo[**], Manuel de León,[***] Manuel Lainz[****]
y Xavier Rivas[*****]

Como es bien sabido, la geometría simpléctica es el formalismo geométrico natural para los sistemas dinámicos hamiltonianos. Las variedades de contacto, en ciertos aspectos, son el análogo en dimensión impar a las variedades simplécticas. Mientras que la dinámica hamiltoniana preserva la energía y la forma simpléctica, la dinámica de los llamados sistemas hamiltonianos de contacto da lugar a disipación de la energía y a una transformación conforme de la uno-forma de contacto. Por ello, la noción de cantidad conservada en diversos contextos ha de ser reemplazada por la de «cantidad disipada».

En esta charla expondré una noción de sistema integrable de contacto. Comentaré la relación con la ecuación de Hamilton–Jacobi para sistemas Hamiltonianos de contacto. Asimismo, presentaré un teorema análogo al de Lioville–Arnold y la construcción de coordenadas de acción-ángulo para estos sistemas.

Agradecimientos Agradezco el apoyo económico recibido mediante las ayudas PID2019-106715GB-C21 y CEX2019-000904-S y el contrato predoctoral PRE2020-093814, financiados por MCIN/AEI/ 10.13039/501100011033.

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An overview about bracket–generating distributions

Javier Martínez Aguinaga*

In this talk we will provide a historical perspective about the study of spaces of bracket-generating distributions from a topological viewpoint. These are subbundles of the tangent space of a manifold satisfying various non-involutive conditions. In recent work with Álvaro del Pino (Utrech University), we have been able to classify certain families of bracket-generating distributions. My goal for the talk will be twofold: I'll try to show the geometry arising behind these results and talk about various interesting open questions in the field.

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VI Congreso de Jóvenes Investigadores

Real Sociedad Matemática Española

León, Febrero de 2023

Bohr-Sommerfeld quantization of b-symplectic toric manifolds

Pau Mir*

Bohr-Sommerfeld quantization uses techniques from symplectic geometry to understand the relation between classical physics and quantum physics. It has been successfully applied in the symplectic case for several compact systems and has been extended to some non-compact ones. For the class of b-symplectic manifolds, the quantization procedure has to be redefined in order to obtain results in finite dimensions. In this talk, I will define the Bohr-Sommerfeld quantization for b-symplectic toric manifolds and show that it coincides with the formal geometric quantization of [1]. In particular, I will prove that its dimension is given by a signed count of the integral points in the moment polytope of the torus action on the manifold. This is joint work with Eva Miranda and Jonathan Weitsman.

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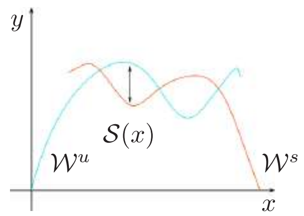
León, Febrero de 2023

Splitting of separatrices for rapidly forced pendulum with a perturbation without first harmonic

Inmaculada Baldomá*, Román Moreno†, Tere M-Seara‡

When an equilibrium point in a planar integrable Hamiltonian system has coincident stable and unstable invariant manifolds —a separatrix— applying a perturbation can lead to its splitting. A usual measure of the splitting for periodic perturbations is the distance, $\mathcal{S}(x)$, between stable ($\mathcal{W}^s$) and unstable manifolds ($\mathcal{W}^u$), of the associated stroboscopic map:



In this work we study the splitting distance of a rapidly perturbed pendulum $H(x,y,t) = \frac{1}{2}y^2 + (\cos(x)-1) + \mu(\cos(x)-1)g\left(\frac{t}{\varepsilon}\right)$ with $g(\tau) = \sum_{|k|>1} g(\tau) = \sum_{|k|>1} g^{[k]}e^{ik\tau}$ a 2π-periodic function and $\mu, \varepsilon \ll 1$. Systems of this kind undergo exponentially small splitting and when $\mu \ll 1$ it is known that the Melnikov function actually gives an asymptotic expression for the splitting function provided $g^{[\pm 1]} \neq 0$. Our study focuses on the case $g^{[\pm 1]} = 0$ and it is motivated by two main reasons. On the one hand the general understanding of the splitting, as current results fail for a perturbation as simple as $g(\tau) = \cos(5\tau) + \cos(4\tau) + \cos(3\tau)$. On the other hand, a study of the splitting of invariant manifolds of tori of rational frequency p/q in Arnold's original model for diffusion leads to the consideration of pendulum-like Hamiltonians with $g(\tau) = \sin\left(p \cdot \frac{t}{\varepsilon}\right) + \cos\left(q \cdot \frac{t}{\varepsilon}\right)$, where, for most $p, q \in \mathbb{Z}$ the perturbation satisfies $g^{[\pm 1]} \neq 0$.

To tackle the problem we use a splitting formula based on the solutions of the inner equation and make use of the Hamilton-Jacobi formalism. In our main result we show that the leading exponentially small term appears at order μ^n, where n is an integer determined exclusively by the harmonics of the perturbation. As was expected, the Melnikov function is in fact not a correct approximation for the splitting in this case.

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VI Congreso de Jóvenes Investigadores

Real Sociedad Matemática Española

León, Febrero de 2023

Periodic perturbation of a 3D conservative flow with 2D heteroclinic connection to saddle-foci

A. Murillo[*], A. Vieiro[*†].

We shall consider a 3D volume-preserving integrable flow with saddle-focus equilibria of types (1,2) and (2,1), and whose 2D unstable/stable invariant manifolds coincide, giving rise to a bubble phase space configuration. Such a system is obtained as a truncation of the normal form of a generic unfolding of a Hopf-Zero bifurcation.

First, we will study the effect of an autonomous perturbation on the previous integrable system. We use the Michelson system, [4], to illustrate the recent results on the asymptotic behaviour of the splitting of the 2D manifolds available in the literature, see [1, 2].

Then, we will consider a non-autonomous periodic perturbation on the previous system. Such a perturbation introduces an additional frequency, which interacts with the intrinsic frequency (which is related to the multiplier of the foci) of the system. In particular, we will focus on the splitting of the 2D stable and unstable invariant manifolds and describe its quasi-periodic asymptotic behaviour. Several illustrations of the perturbed flow system and the behaviour of the splitting of the invariant manifolds will be shown.

Some final numerical simulations using the Michelson map, [3], will serve to discuss the analogies and differences between the dynamics of the map and the previous periodically forced integrable flow.

Acknowledgements A. Murillo and A. Vieiro are supported by grants PID2019-104851GB-I00 (Spain) and 2017-SGR-1374 (Catalonia).

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VI Congreso de Jóvenes Investigadores

Real Sociedad Matemática Española

León, Febrero de 2023

Hamiltonian facets of classical gauge theories on E-manifolds

A. Pablo Nicolás*

Symplectic geometry and classical mechanics intersect on what is known as Hamiltonian mechanics, which are especially well suited for the study of symmetries in physical problems. The theory of reduction inagurated by Marsden and Weinstein [2] can be seen a natural generalization of Noether's theorem for conserved quantities. In this setting, classical gauge theories describe the motion of a charged particle under a gauge field. The phase space of such systems is a symplectic leaf of the cotangent bundle T^*B, where $\tau\colon B \longrightarrow M$ is a G-bundle. Weinstein [7] realized B as the adjoint bundle of a fibre bundle $\pi\colon P \longrightarrow M$, and proved that the equations of movement of a charged particle are Hamiltonian. Later, Montgomery realized the phase spaces of Weinstein as leaves of a bigger Poisson space [5] and gave a Poisson generalization of the *minimal coupling procedure* in electromagnetism. Under Weinstein's observation, a classical gauge theory is equivalently given by a free and proper action of a Lie group G on M. This observation, besides identifying classical gauge theories within the theory of symplectic reduction, justifies why gauge theories are also coined in the literature under the name of *cotangent bundle reduction* [1].

In this talk, we extend the results of Weinstein and Montgomery to the setting of E-manifolds. These structures contain manifolds with boundary, with corners, b-manifolds and regular foliations, and model configuration spaces for particles moving under constraints. E-Manifolds were introduced by Nest and Tsygan [6] and investigated in depth by Miranda and Scott [4]. After reviewing the theory of E-manifolds, we extend the language of gauge fields to include singularities encoded in an E-structure. Following the scheme inaugurated by Weinstein, we show the existence of a universal model for a particle interacting with an E-gauge field. In addition, we generalize the description of phase spaces in Yang-Mills theory as Poisson manifolds and their minimal coupling procedure, as shown by Montgomery, for base manifolds endowed with an E-structure. In particular, the reduction at coadjoint orbits and the shifting trick are extended to this framework. We show that Wong's equations, which describe the interaction of a particle with a Yang-Mills field, become Hamiltonian in the E-setting.

This new description allows us to approach problems in dynamical systems, such as describing the behaviour of trajectories near infinity, using geometric techniques. This is joint work with Pau Mir and Eva Miranda [3].

Acknowledgments The author has been supported by a collaboration grant of the Spanish Ministry of Education and by a Fundació Cellex grant within the CFIS programme.

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León, Febrero de 2023

A degenerate Arnold Diffusion Mechanism in the Restricted 3 Body Problem

Jaime Paradela,* Marcel Guardia† Tere M. Seara‡

A major question in dynamical systems is to understand the mechanisms driving global instability in the 3 Body Problem (3BP), which models the motion of three bodies under gravitational interaction. The 3BP is called *restricted* if one of the bodies has zero mass and the other two, the primaries, have strictly positive masses m_0, m_1. In this limit problem the goal is to describe the motion of the massless body. We consider the Restricted Planar Elliptic 3 Body Problem (RPE3BP) where the massive bodies revolve one around each other in Keplerian ellipses. We prove that the RPE3BP exhibits topological instability: *for any values of the masses* m_0, m_1 (except $m_0 = m_1$), we build orbits along which the angular momentum of the massless body experiences an arbitrarily large variation provided the eccentricity of the orbit of the massive bodies is small enough.

In order to prove this result we show that a *degenerate Arnold Diffusion Mechanism* takes place in the RPE3BP. More concretely, we build a transition chain of periodic orbits located in a topological Normally Hyperbolic Invariant Cylinder which is foliated by fully resonant invariant tori. The setting in which we construct the Arnold Diffusion mechanism displays many features of the so called *a priori stable* case.

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León, Febrero de 2023

A simple proof of Novikov's theorem

Samuel Ranz*

When a compact three-manifold admits a taut foliation, the inclusion map induces an homotopy homomorphism between the fundamental group of the leaves and that of the manifold. This result is known as Novikov's theorem. In this talk we discuss how to simplify the proof of this result using approximately holomorphic techniques on foliations.

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León, Febrero de 2023

Holomorphic h–principles over Stein manifolds

Guillermo Sánchez Arellano*

A key feature of the Differential Geometry of the last 70 years has been to find the limits where a geometric problem begins to be directed by the rules of Differential Topology. There were several results in that direction ([8], [9], [5], [6], [7], [4]) and they where encapsulated by M. Gromov in a general framework introducing the now well-known *homotopy principle* [3].

Supose that some geometric objects of your interest (e.g. symplectic forms, contact or Engel structures) can be understood as sections of a bundle $E \to B$ that satisfy some differential conditions. That conditions define a differential relation, i.e. a subset $\mathcal{R}$ of the r–jet bundle $E^{(r)}$ of E. It is said that the problem satisfy the h–principle if the space of formal solutions (i.e. sections of $\mathcal{R} \to B$) is homotopically equivalent to the space of holonomic solutions (i.e. sections of $E \to B$ whose r–jet lie into $\mathcal{R}$).

Finding an h–principle reduces the (a priori) difficult geometric problem of finding sections with differential conditions to a purely topological one. There are several standard techniques to find h–principles in the smooth category and they have been particularly useful in Contact and Symplectic Topology.

Nevertheless there are very few results in the holomorphic category. The first steps in that direction have been given by the work of F. Forstnerič and Slapar in [2] and by F. Forstnerič in [1]. In this talk we will broaden that results to general differential relations in holomorphic jet spaces of holomorphic bundles over Stein manifolds. This is Joint work with Professor Luis Giraldo of Facultad de Ciencias Matemáticas UCM.

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Geometría Convexa

Organizadores: Francisco Marín y Jesús Yepes

Martes 7 de Febrero
Moderadores: *Bernardo González Merino y Antonio Cañete*

10:00 *Brunn-Minkowski inequality for θ-convolution bodies via Ball's bodies*, David Alonso Gutiérrez

10:30 *A unimodular theory of reduced and complete convex bodies*, Ansgar Freyer

18:15 *Large signed subset sums*, Bernardo González Merino

18:45 *Geometric inequalities involving different diameter definitions*, Mia Runge

19:15 *Fórmulas tubulares*, Juan Andrés Trillo

Miércoles 8 de Febrero
Moderadores: *Eduardo Lucas y David Alonso-Gutiérrez*

10:00 *Prescribed mean curvature graphs in sub-Finsler Heisenberg groups*, Julián Pozuelo

10:30 *Proximity and flatness bounds via a volume argument*, Stefan Kuhlmann

11:30 *Discrete variations of the Brunn-Minkowski inequality*, Eduardo Lucas

11:55 *Ellipsoids in convex bodies of a given asymmetry*, Florian Grundbacher

12:20 *A discrete approach to Zhang's inequality*, Javier Martín Goñi

12:45 *Affine subspace concentration conditions for centered polytopes*, Christian Kipp

13:10 *The Cheeger set of a rotationally symmetric planar convex body*, Antonio Cañete

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A discrete approach to Zhang's inequality

D. Alonso-Gutiérrez,* E. Lucas Marín,† Javier Martín Goñi (speaker).‡

Given a convex body $K \subset \mathbb{R}^n$, i.e., a compact convex set with non empty interior, its polar projection body $\Pi^* K$ is the unit ball of the norm given by

$$\|x\|_{\Pi^* K} = \|x\|_2 \mathrm{vol}_{n-1}\left(P_{x^\perp}(K)\right),$$

where $P_{x^\perp}(K)$ denotes the orthogonal projection of K onto $x^\perp$.

Zhang's inequality [1], proves that among all the n-dimensional convex bodies, the affine invariant quantity $(\mathrm{vol}_n(K))^{n-1}\,\mathrm{vol}_n(\Pi^* K)$ is minimized if and only if K is a simplex (i.e. the convex hull of $n+1$ affinely independent points). Thus, for any convex body $K \subset \mathbb{R}^n$,

$$\frac{\binom{2n}{n}}{n^n} \le (\mathrm{vol}_n(K))^{n-1}\,\mathrm{vol}_n(\Pi^* K).$$

Several proofs and extensions of this inequality have been given in the last decades. In this talk, we will obtain an approach to Zhang's inequality in the discrete setting, where we will consider discrete measures instead of the lebesgue measure.

This is part of a joint work with David Alonso-Gutiérrez and Eduardo Lucas.

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Large signed subset sums

G. Ambrus, [*] Bernardo González Merino (speaker). [†]

In this talk, we consider for parameters n, d, k such that $2 \leq d \leq n$, $2 \leq k \leq n$, the minimum number $c(n, d, k)$ such that from any set of n unit vectors from $\mathbb{R}^d$, we can select k of them and signs, so that their signed sum has norm at least $c(d, n, k)$.

This question is partly motivated from different topics and viewpoints: it partly solves questions about lower bounds for the radius of the sum of convex bodies, it is the dual to vector sum minimization and balancing questions, and some particular cases connected to questions about p-frame energies.

We give asymptotic sharp estimates for the general case $c(d, n, k)$. We give a better asymptotic estimate in the case $c(d, n, 2)$, and give sharp bounds in the planar case $c(2, n, k)$.

This is a joint work with Gergely Ambrus.

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Fórmulas tubulares

Juan Andrés Trillo.*

Una *valoración diferenciable* en una variedad riemanniana M es un funcional

$$A \longmapsto \int_{N(A)} \omega + \int_A \eta,$$

donde $A \subset M$ es una subvariedad con esquinas, $\omega \in \Omega^{2n-1}(SM)$, $\eta \in \Omega^{2n}(M)$ y $N(A)$ es el ciclo normal de A. Sea $M = \mathbf{C}P_\lambda^n$ la variedad de Kähler de dimensión compleja n, de curvatura holomorfa constante 4λ, completa y simplemente conexa. Al espacio conformado por las valoraciones de $\mathbf{C}P_\lambda^n$ invariantes por el grupo de isometrías, lo denotamos por $\mathcal{V}_\lambda^n$ que es un espacio vectorial de dimensión finita.

Nuestro principal resultado es el cálculo de $\mu(A_t)$, para cualquier valoración μ y cualquier subvariedad con esquinas A, siendo A_t el tubo de radio $t \geq 0$ alrededor de A. El resultado es un polinomio trigonométrico en t cuyos coeficientes son valoraciones invariantes evaluadas en A. Para ello, traducimos el problema al análisis. Variando el radio del tubo obtenemos una función diferenciable que, al derivarla, induce el operador $\tilde{\Lambda}_\lambda$, conocido como *operador derivada*. Tal construcción nos permite plantear el sistema de ecuaciones diferenciales ordinarias de coeficientes constantes $\partial_t(\mu)_t = \tilde{\Lambda}_\lambda \mu$.

Para resolver el sistema anterior nos basamos en una sl_2-representación subyacente a nuestro espacio de valoraciones y su consecuente descripción en términos de *polinomios homogéneos*. A su vez, encontramos todos los valores propios y sus correspondientes subespacios propios del operador $\tilde{\Lambda}_\lambda$. En particular, caracterizamos núcleo de $\tilde{\Lambda}_\lambda$, las llamadas *valoraciones primitivas*, conocidas por ser invariantes por el flujo de Reeb. Por último, damos una descripción de la característica de Euler en términos de las valoraciones primitivas.

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The Cheeger set of a rotationally symmetric planar convex body

Antonio Cañete *

Let Ω be a k-rotationally symmetric planar convex body, and let C_Ω be the *Cheeger set* associated to Ω (that is, among all the subsets of Ω, C_Ω is the one minimizing the quotient of the perimeter over the enclosed area).

In this talk, we will describe some properties of C_Ω. In particular, we will show that C_Ω is k-rotationally symmetric.

Additionally, by introducing previously the notions of *dots* and *edges* of Ω, we will prove that the boundary of C_Ω touches all the edges of Ω (in other words, Ω is a Cheeger-regular set, in view of a definition from a paper by Kawohl and Lachand-Robert).

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Proximity and flatness bounds via a volume argument

M. Celaya,[*] Stefan Kuhlmann (speaker),[†] J. Paat,[‡] R. Weismantel.[§]

We present novel upper bounds on two concepts from integer linear optimization: The first is the distance between a vertex solution of the LP-relaxation and an optimal solution of the integer program, the proximity. The second is the lattice width of lattice-free polyhedra. These bounds rely on a volume argument. A key technique for both problems concerns the area of a convex polygon P: If P is contained in its polar after a 90° rotation, then the area of its polar is at least three. This is based on joint work with Marcel Celaya, Joseph Paat, and Robert Weismantel.

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A unimodular theory of reduced and complete convex bodies

G. Codenotti, [*] Ansgar Freyer (speaker). [†]

In Euclidean geometry, a convex body of K is called complete, if there is no proper superset of K with the same diameter. Similarly, K is called reduced, if there is no proper subset of K with the same width. These concepts have also been studied extensively in other geometries, such as the sphere, hyperbolic space or Minkwoski spaces.

In a joint work with Giulia Codenotti we investigate these notions when the width and diameter of a convex body $K \subset \mathbb{R}^d$ are measured with respect to the integer lattice. In the talk, we will derive structural results on the reduced and complete bodies in this setting, present methods to construct examples and discuss applications to problems around the flatness constant in integer programming.

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Affine subspace concentration conditions for centered polytopes

A. Freyer, * M. Henk, † Christian Kipp (speaker).‡

Given an n-dimensional polytope P with $0 \in P$, the cone volume corresponding to a facet $F_i \subset P$ is the volume of the cone with apex 0 and base F_i. We say that P satisfies the subspace concentration condition if for every d-dimensional linear subspace L, the total cone volume of the facets with outer normal vectors in L is at most $\frac{d}{n} \cdot \mathrm{vol}(P)$. The subspace concentration condition plays a central role in the context of the (discrete) logarithmic Minkowski problem, which asks for a characterization of cone-volume measures of polytopes. This problem is, in the general (non-even) case, still open.

Recently, an affine version of the subspace concentration condition was shown by Wu to hold for certain lattice polytopes: In the affine setting, Wu replaced the outer normal vectors by the vertices of the polar $P^\star$ and the linear subspace L by an affine subspace A, leading to a sharper inequality in certain cases. In this talk, we discuss an extension of this result to arbitrary centered polytopes as well as some open questions concerning the equality case. This is joint work with Ansgar Freyer and Martin Henk.

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Discrete variations of the Brunn-Minkowski inequality

A. Freyer,* M. A. Hernández Cifre,† Eduardo Lucas Marín (speaker),‡ J. Yepes Nicolás.§

In this talk we will explore several discrete versions of the Brunn-Minkowski inequality, i.e.,

$$\mathrm{vol}\big((1-\lambda)K+\lambda L\big)^{1/n} \geq (1-\lambda)\mathrm{vol}(K)^{1/n} + \lambda\mathrm{vol}(L)^{1/n}$$

for non-empty compact sets $K, L \subset \mathbb{R}^n$ and $\lambda > 0$, as well as some of its well-known variations, such as its extension to the L_p setting. For this, we will make use of the discrete measure given by the *lattice point enumerator*, $\mathrm{G}_n(K) = |K \cap \mathbb{Z}^n|$ (where $|\cdot|$ is the cardinality measure).

We will introduce some of the difficulties that arise in this context, and we will propose various solutions that have been fruitful recently. In particular, we will discuss the strategy of modifying the sets involved in a controlled manner, as well as one based on incorporating factors in the inequality. Furthermore, we will relate these discrete versions with their traditional counterparts.

This talk is based on joint work with A. Freyer, M. A. Hernández Cifre and J. Yepes Nicolás.

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Prescribed mean curvature graphs in sub-Finsler Heisenberg groups

G. Giovannardi,[*] A. Piamonti,[†] Julián Pozuelo (speaker),[‡] S. Verzellesi.[§]

This work is in colaboration with Gianmarco Giovannardi, Andrea Pinamonti and Simone Verzellesi. We consider the n-Heisenberg group $\mathbb{H}^n$ and a left-invariant norm $|\cdot|$ in the horizontal distribution. Let $\Omega \subset \mathbb{R}^{2n}$ be a bounded open set, $H \in L^\infty(\Omega)$, $F \in L^1(\Omega, \mathbb{R}^{2n})$ and $u \in W^{1,1}(\Omega)$. We consider the functional

$$u \mapsto \int_\Omega |\nabla u + F|_* dxdy + \int_\Omega Hudxdy,$$

where $|\cdot|_*$ denotes the dual norm of $|\cdot|$. This functional is the prescribed mean curvature functional for t-graphs in the sub-Finsler group $\mathbb{H}^n$. When we fix a boundary datum $\varphi \in C^2(\partial\Omega)$, a solution to the Dirichlet problem is a minimizer of the functional such that $u = \varphi$ in $\partial\Omega$. We shall prove that under suitable regularity assumptions on the data, there exists a Lipschitz solution to the Dirichlet problem for the prescribed mean curvature equation.

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Ellipsoids in convex bodies of a given asymmetry

Florian Grundbacher.*

In "Ellipsoids" of maximal volume in convex bodies Keith Ball proved a general bound on the volume of k-dimensional ellipsoids in n-dimensional convex bodies in relation to their John ellipsoid. A stronger bound is known in the symmetric case. Our goal was to connect these results by establishing a bound depending on the John asymmetry s_0. We could prove a tight bound for all k and all asymmetry values $s_0 \notin (1, 1 + 2/n)$, and characterize the equality cases.

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Brunn-Minkowski inequality for θ-convolution bodies via Ball's bodies

J. Martín Goñi, [*] David Alonso-Gutiérrez (speaker). [†]

In this talk we will consider the problem of finding the best possible function $\varphi_n : [0,1] \to \mathbb{R}$ such that for any pair of convex bodies $K, L \subseteq \mathbb{R}^n$ the following Brunn-Minkowski type inequality is satisfied:

$$|K +_\theta L|^{\frac{1}{n}} \geq \varphi_n(\theta)(|K|^{\frac{1}{n}} + |L|^{\frac{1}{n}}),$$

where $K +_\theta L$ denotes the θ-convolution body of K and L, defined as

$$K +_\theta L = \{x \in K + L : |K \cap (x - L)| \geq \theta \max_{x_0 \in \mathbb{R}^n} |K \cap (x_0 - L)|\}.$$

We will provide the best possible function in the range $\left(\frac{3}{4}\right)^n \leq \theta \leq 1$, characterizing the equality cases. The result will be obtained as a consequence of a sharp inclusion result of Ball bodies of an α-concave function into its super-level sets.

This is a joint work with Javier Martín Goñi.

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Geometric inequalities involving different diameter definitions

Mia Runge.*

We study Blaschke-Santaló diagrams for the inradius, circumradius and diameter. When considering non-symmetric gauges there are several ways to define the diameter of a convex body which correspond to different sym- metrizations of the gauge. One of these diameters is examined in detail. By proving new inequalities and properties we show that in the planar case the union over all possible gauges of the diagrams coincides with the diagram of a triangle.

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Geometría Diferencial

Organizadores: Adela Latorre, Alberto Rodríguez y Erik Sarrión

Lunes 6 de Febrero
Moderador: *Erik Sarrion-Pedralva*

12:30 *The Audibility problem from a non-compact point of view using generalized Heisenberg groups*, Jose Manuel Fernández-Barroso.

13:00 *High energy eigenfunctions on flat tori*, Alba García-Ruiz.

18:15 *Prescribing values of a complete minimal surface whose Gauss map misses two point*, Ildefonso Castro-Infantes.

18:45 *Superficies de Bryant completas*, Jorge Hidalgo Calderón.

Martes 7 de Febrero
Moderadores: *Alberto Rodríguez-Vázquez y Erik Sarrion-Pedralva*

10:00 *Superficies de curvatura lineal de Weingarten predeterminada*, Irene Ortiz

10:30 *Inestabilidad de Plateau-Rayleigh para λ-solitones*, Antonio Bueno

11:30 *Overdetermined elliptic problems in the sphere*, Jing Wu

12:00 *Métricas curvatura homogéneas críticas para funcionales cuadráticos de la curvatura*, Sandro Caeiro Oliveira

12:30 *Técnicas de análisis geométrico para un problema variacional en geometría (pseudo-)Finsler*, Fidel F. Villaseñor

13:00 *El problema de Zermelo y su aplicación a la modelización de incendios forestales*, Enrique Pendás Recondo

18:15 *Acciones de grupos finitos y nilvariedades: propiedad Jordan y rigidez*, Jordi Daura Serrano

18:45 *Desigualdad de tipo Basmajian para superficies Riemannianas con frontera*, David Fisac Camara

Miércoles 8 de Febrero
Moderadora: *Adela Latorre*

10:00 *Estructuras geométricas en el fibrado tangente generalizado*, Pablo Gómez-Nicolás

10:30 *Futaki invariants for the Hull-Strominger system*, Raul Gonzalez Molina

11:30 *Solitones de Ricci homogéneos y grupos nilpotentes de Iwasawa*, Víctor Sanmartín-López

12:00 *Upper and lower bounds on the filling radius*, A. Manuel Cuerno

12:30 *Quantizability of manifold invariants*, Ángel González-Prieto

Métricas curvatura homogéneas críticas para funcionales cuadráticos de la curvatura

M. Brozos Vázquez [*], S. Caeiro Oliveira [**], E. García Río[**]

Una variedad pseudo-riemanniana (M, g) es k-curvatura homogénea si, para cada par de puntos $p, q \in M$, existe una isometría linear $\phi_{pq} : T_pM \to T_qM$ satisfaciendo $\phi_{pq}^* \nabla^i R_q = \nabla^i R_p$ para todo $0 \leq i \leq k$. Si $k = 0$, decimos que la variedad es curvatura homogénea. Este tipo de variedades generaliza las homogéneas, ya que una variedade homogénea es curvatura homogénea (de hecho, es k-curvatura homogénea para todo k), pero el recíproco no es cierto.

Las variedades Einstein son métricas críticas para el funcional de Hilbert-Einstein restringido a variaciones de volumen constante. Dicho funcional se define a partir de la integral de volumen de la curvatura escalar de la variedad. Dado que la curvatura escalar es el único invariante escalar de la curvatura de orden uno, para generalizar estas métricas podemos tomar funcionales introducidos por medio de acciones que involucran invariantes escalares de orden superior.

En esta charla clasificamos las variedades curvatura homogéneas de dimensión tres modeladas en espacios simétricos que son críticas para algún funcional cuadrático de la curvatura y aquellas que proporcionan soluciones para las teorías de gravitación masiva. Un hecho a destacar es que la estructura en todas las soluciones es una onda de Brinkmann o un espacio-tiempo de Kundt, proporcionando estos últimos métricas no Einstein que son críticas para todos los funcionales cuadráticos.

Agradecimientos Los autores han sido financiados por los proyectos PID2019-105138GB-C21 / AEI/ 10.13039/ 501100011033 (Spain) y ED431C 2019/10, ED431F 2020/04 (Xunta de Galicia, Spain).

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Inestabilidad de Plateau-Rayleigh para λ-solitones

Antonio Bueno[*] Rafael López[**] Irene Ortiz[*]

Se conoce como *fenómeno de Plateau-Rayleigh* al hecho de que un flujo de agua de radio r que cae verticalmente se rompe en gotas del mismo volumen y menor área de superficie cuando la longitud de la corriente de agua es mayor que $2\pi r$. Este hecho fue observado experimentalmente por Plateau y demostrado de forma teórica por Rayleigh.

Desde el punto de vista de la Geometría Diferencial, este problema se puede formular de la siguiente forma: dado $r > 0$, consideramos C_r^L un trozo compacto de longitud L del cilindro de curvatura media constante (CMC) $H = 1/(2r)$. ¿Existe L_0 tal que si $L > L_0$, C_r^L deja de ser estable?

La idea principal para formular este problema es el hecho de que las superficies CMC son puntos críticos del funcional área bajo variaciones de soporte compacto que preservan el volumen. Posteriormente, se estudia el operador de estabilidad y su forma cuadrática asociada, con el objetivo de encontrar uan función test adecuada con la que conseguir la deseada inestabilidad.

En este sentido, podemos considerar puntos críticos de los funcionales área y volumen cuando éstos se calculan respecto de los elementos área y volumen medidos con una *densidad*. Se define el espacio Euclídeo con densidad $\phi \in C^\infty(M)$ cuando en $\mathbb{R}^3$ se consideran los ementos de área y volumen *con peso*

$$dA_\phi = e^\phi dV, \qquad dV_\phi = e^\phi dV.$$

En este trabajo consideramos la densidad $\phi(x) = \langle x, \mathbf{v}\rangle,\ |\mathbf{v}| = 1$. Una superficie orientada M en $\mathbb{R}^3$ es punto crítico del área con peso respecto de variaciones que preservan el volumen con peso si su curvatura media H_M satisface

$$H_M = \langle N, \mathbf{v}\rangle + \lambda, \qquad \lambda \in \mathbb{R}, \tag{1}$$

donde N es su aplicación de Gauss. Diremos que M es un λ-solitón si su curvatura media viene dada por (1). Destacar que en el caso $\lambda = 0$ recuperamos los *solitones de traslación del flujo por curvatura media*.

El objetivo de esta charla es obtener un resultado de inestabilida del tipo Plateau-Rayleigh para los λ-solitones de tipo cilíndrico, esto es, para superficies parametrizadas por

$$x(s,t) = \alpha(s) + t\mathbf{w}, \qquad |\mathbf{w}| = 1,$$

donde $\alpha(s)$ es una curva parametrizada por el arco contenida en un plano ortogonal a $\mathbf{w}$. Para tal fin, aprovecharemos la clasificación del primer y tercer autores de los λ-solitones de tipo cilíndrico [2], estudiaremos el operador de estabilidad asociado y propondremos funciones test adecuadas para lograr la inestabilidad deseada.

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VI Congreso de Jóvenes Investigadores

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León, Febrero de 2023

Superficies de curvatura lineal de Weingarten predeterminada

Antonio Bueno [*] Irene Ortiz[†]

Sean $\Phi \in \mathcal{C}^1(\mathbf{S}^2)$, $a, b \in \mathbf{R}$. Una superficie orientada M en el espacio euclídeo 3-dimensional $\mathbf{R}^3$ se dice que tiene curvatura lineal de Weingarten predeterminada si sus curvaturas media H y de Gauss K satisfacen (1) $2aH + bK = \Phi(N)$, donde $N : M \to \mathbf{S}^2$ es su aplicación de Gauss. Estas superficies generalizan a otras que han sido ampliamente estudiadas en la literatura, entre las que destacamos:

- $b = 0$, $\Phi = c$, superficies de curvatura media constante.
- $a = 0$, $\Phi = c$,superfices de curvatura de Gauss constante.
- $ab \neq 0$, $\Phi = c$, superficies lineales de Weingarten.
- $b = 0$, $\Phi(X) = \langle X, e_3 \rangle$, solitones de traslación del flujo por curvatura media.

La definición de esta clase de superficies está motivada por el problema de Minkowski, el cual plantea la existencia y unicidad de un ovaloide cuya curvatura de Gauss satisface $K = \Phi(N)$.

En el caso de curvatura media predeterminada satisfaciendo $H = \Phi(N)$, Alexandrov, Pogorelov, Hartman y Wintner, entre otros, y más recientemente Gálvez y Mira estudiaron la existen- cia y unicidad de ovaloides. El primer autor, junto con Gálvez y Mira, comenzó a desarrollar la teoría global de superficies de curvatura media predeterminada, haciendo especial énfasis en la clasificación de las superficies rotacionales.

El objetivo de esta charla es doble. Por una parte, iniciar el de- sarrollo de una teoría global de superficies satisfaciendo (1), comenzando por el estudio de las superficies rotacionales. Por otra parte, completar y corregir la clasificación de superficies lineales de Weingarten rotacionales en los casos elíptico e hiperbólico.

La herramienta principal usada para tal fin es el análisis de las soluciones de un sistema de primer orden autónomo y no lineal que satisfacen las coordenadas de la curva perfil de una superficie de rotación.

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VI Congreso de Jóvenes Investigadores

Real Sociedad Matemática Española

León, Febrero de 2023

Prescribing values of a complete minimal surface whose Gauss map misses two point

I. Castro-Infantes*

The study of the (generalized) Gauss map of a conformal minimal immersion is an important topic in the theory of minimal surfaces. Alarcón, Fernández, and López [1] proved that for any open Riemann surface M, there exists a complete conformal minimal immersion $X\colon M \to \mathbb{R}^n$ whose generalized Gauss map G_X is nondegenerate and fails to intersect n hyperplanes in general position; the number n here is the maximum possible. We will show in this talk that one can prescribe the values of such an immersion on a closed discrete subset of M.

Let M be an open Riemann surface and let $\Lambda \subset M$ be a closed discrete subset. We would prove the existence of complete conformal minimal immersions $M \to \mathbb{R}^n$, $n \geq 3$, with prescribed values on Λ and whose generalized Gauss map $M \to \mathbb{CP}^{n-1}$, $n \geq 3$, avoids n hyperplanes of $\mathbb{CP}^{n-1}$ located in general position. In case $n = 3$, we obtain complete nonflat conformal minimal immersions whose Gauss map $M \to \mathbb{S}^2$ omits two (antipodal) values of the sphere.

The previous result may be found on [2]

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VI Congreso de Jóvenes Investigadores
Real Sociedad Matemática Española
León, Febrero de 2023

Upper and lower bounds on the filling radius

A. Manuel Cuerno*

A Riemannian manifold M can be isometrically embedded into $L^\infty(M)$ via the Kuratowski embedding, sending each point $p \in M$ to its distance function $\mathrm{dist}_p(\cdot) \in L^\infty(M)$. In 1983, Gromov defined the Filling Radius of M as the infimum of the $\epsilon > 0$ for which the fundamental class vanishes in the ϵ–neighborhood of the image of M in $L^\infty(M)$. In other words, $\mathrm{FillRad}(M)$ measures the minimal size of the $(n+1)$–dimensional hole that captures a n-dimensional manifold.

In this talk, I will give a brief introduction to Gromov ideas and will provide some new upper and lower bounds on the Filling Radius in terms on the curvature and other metric invariants that have been obtained in collaboration with my advisor Luis Guijarro.

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VI Congreso de Jóvenes Investigadores
Real Sociedad Matemática Española
León, Febrero de 2023

Acciones de grupos finitos y nilvariedades: propiedad Jordan y rigidez

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La teoría de grupos finitos de transformación tiene por objetivo estudiar las simetrías de espacios topológicos y variedades mediante acciones de grupos finitos. Recordemos que una acción efectiva de un grupo finito G en una variedad M es un morfismo de grupos inyectivo $G \longrightarrow \mathrm{Aut}(M)$, donde $\mathrm{Aut}(M)$ puede denotar $\mathrm{Diff}(M)$ si consideramos acciones diferenciables o $\mathrm{Homeo}(M)$ si solo consideramos acciones continuas.

Un problema fundamental en la teoría de grupos de transformación finitos es determinar qué grupos finitos pueden actuar de forma efectiva en una una variedad cerrada (compacta y sin frontera) M dada. En general, este problema es extremadamente difícil y por tanto tratamos de simplificarlo estudiando qué restricciones deben cumplir los grupos finitos que actúan en M. Por ejemplo, diremos que un grupo $\mathcal{G}$ es Jordan si existe una constante C tal que todo subgrupo finito G de $\mathcal{G}$ tiene un subgrupo abeliano $A \leq G$ tal que $[G : A] \leq C$. En la década de los noventa Étienne Ghys conjeturo que el grupo $\mathrm{Diff}(M)$ de una variedad cerrada M es siempre Jordan. Desde entonces se han encontrado ejemplos de variedades que cumplen la conjetura ([1, 2, 3]), pero también se han encontrado contraejemplos ([4]). Todavía no se sabe ningún procedimiento general para determinar si una variedad cerrada tiene grupo de difeomorfismos Jordan.

También podemos estudiar la pregunta inversa: ¿podemos determinar una variedad cerrada M si sabemos que grupos actúan en ella? Por ejemplo, un resultado clásico nos dice que si el toro T^n actúa en una variedad cerrada M de dimensión n entonces $M \cong T^n$. Una pregunta análoga para acciones de grupos finitos es si una variedad cerrada M de dimensión n que admite acciones efectivas de grupos de la forma $(\mathbb{Z}/a\mathbb{Z})^n$ para $a \in \mathbb{N}$ arbitrariamente grande cumple que $M \cong T^n$. En [5], esta pregunta se responde de forma afirmativa si suponemos que M admite una aplicación de grado distinto de cero a T^n y $\pi_1(M)$ es virtualmente resoluble.

En esta charla daremos nuevos ejemplos de variedades cerradas M con $\mathrm{Diff}(M)$ Jordan, como las nilvariedades (cocientes N/Γ donde N es un grupo de Lie nilpotente simplemente conexo y Γ es un retículo de N). También discutiremos como podemos generalizar los resultados de [5] a variedades M que admiten una aplicación de grado distinto de cero hacia una nilvariedad. Esto nos llevará a introducir el concepto de acción iterada de grupos y nos motivará a construir una teoría de acciones iteradas de grupos finitos análoga a la teoría de grupos finitos de transformación.

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VI Congreso de Jóvenes Investigadores
Real Sociedad Matemática Española
León, Febrero de 2023

The Audibility problem from a non-compact point of view using generalized Heisenberg groups

J.M. Fernández-Barroso*

The Spectral Geometry is a field inside the Riemannian Geometry whose main goal is to relate the geometry of a Riemannian manifold and the eigenvalues of a given operator, in our case the Laplace–Beltrami operator. In this context, two Riemannian manifolds are said to be isospectral if there exists a unitary operator intertwining their Laplace–Beltrami operators. When the Riemannian manifolds are compact, this definition is the same that both manifolds have the same spectrum of the Laplace–Beltrami operator. A geometric property cannot be *heard* (or it is inaudible) if one can find a pair of isospectral manifolds where that property differs.

The compact setting has been well studied in several papers where many properties are proved to be inaudible. But there are only a few papers approaching the non compact setting. Being a g.o. space was proved by Carolyn Gordon to be an inaudible property, at least in the non compact case. In this talk, we want to clarify the difference between the compact and the non compact cases using generalized Heisenberg groups, and we use them to establish the inaudibility of the weak symmetry and the commutativity in the non compact case. Finally, we conclude the talk introducing some interesting questions relating the compact and the non compact setting.

Agradecimientos This is a joint work with Teresa Arias-Marco.

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León, Febrero de 2023

Desigualdad de tipo Basmajian para superficies Riemannianas con frontera

David Fisac Camara*

La identidad de Basmajian [1] es un resultado clásico en superficies hiperbólicas con frontera que permite medir la longitud de la frontera de una superficie solo en función de su ortoespectro, i.e. las longitudes de los arcos geodésicos con puntos extremos en la frontera de la superfície.

A partir de los trabajos de Lim (ver [2]), presentamos una desigualdad para una familia de grafos métricos que nos permitirá concluir en una desigualdad de tipo Basmajian para métricas Riemannianas con entropía positiva, dónde la entropía se refiere a la entropía volumétrica para superficies Riemannianas compactas (o crecimiento volumétrico asimptótico).

Finalmente, hablaremos de aplicaciones de esta desigualdad para algunas familias de variedades estudiadas.

Agradecimientos Esta charla se basa en trabajo conjunto con Florent Balacheff.

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VI Congreso de Jóvenes Investigadores

Real Sociedad Matemática Española

León, Febrero de 2023

High energy eigenfunctions on flat tori

Alba García-Ruiz*

A well-known link between solutions to the Helmholtz equation $\Delta h + h = 0$ and eigenfunctions of the Laplacian on a closed Riemannian manifold, $\Delta_g u + \lambda^2 u = 0$, is that the local behaviour of a sequence of high-energy eigenfunctions (say, $\lambda^2 \to \infty$) defines a bounded Helmholtz solution, after a suitable rescaling. Conversely, every solution to Helmholtz can be locally realized by an approximate eigenfunction of any large enough energy, on scales determined by this energy.

A powerful refinement of the latter fact is what we call the inverse localization principle: if, roughly speaking, the degeneracy of the high-energy eigenvalues is large enough, one can replace the quasimodes by bona fide eigenfunctions.

In this joint work with A. Enciso and D. Peralta-Salas, we consider the question of when the Laplace eigenfunctions on an arbitrary flat torus $\mathbb{T}_\Gamma = \mathbb{R}^d/\Gamma$ are flexible enough to approximate, over the natural length scale of order $1/\lambda$ an arbitrary solution of the Helmholtz (i.e. when $\mathbb{T}_\Gamma$ satisfy the inverse localization property).

Finally, as a consequence of this fact, we show that when $\mathbb{T}_\Gamma$ satisfy the inverse localization property, Laplace eigenfunctions exhibit an extremely flexible behaviour over scales of order $1/\lambda$. In particular, there are eigenfunctions of arbitrarily high energy that exhibit nodal components diffeomorphic to any compact hypersurface of diameter $\mathcal{O}(1/\lambda)$.

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VI Congreso de Jóvenes Investigadores

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León, Febrero de 2023

Estructuras geométricas en el fibrado tangente generalizado

P. Gómez-Nicolás,[*] F. Etayo[*], R. Santamaría[**]

La geometría generalizada tiene como base el estudio del fibrado tangente generalizado $\mathbb{T}M := TM \oplus T^*M \to M$, definido como la suma de Whitney del fibrado tangente y cotangente de una variedad. El estudio de este fibrado fue propuesto por N. Hitchin en 2003 [1] y desarrollado posteriormente por M. Gualtieri [2].

Igual que ocurre con el fibrado tangente de una variedad, se pueden definir diversas estructuras geométricas en el fibrado $\mathbb{T}M$, tales como métricas, estructuras simplécticas, estructuras complejas, etc. Estas estructuras las denominamos estructuras geométricas generalizadas. Hay algunas estructuras geométricas generalizadas que nacen de manera canónica: es el caso de una métrica pseudo-Riemanniana, una estructura simpléctica y una estructura paracompleja [3]. Por otra parte, otras estructuras en $\mathbb{T}M$ se pueden inducir a partir de estructuras geométricas definidas sobre la variedad. Según el tipo de estructura sobre la variedad que se tenga, se podrán inducir diferentes estructuras generalizadas en $\mathbb{T}M$ [4, 5]. La motivación original de M. Gualtieri al trabajar sobre el fibrado tangente generalizado fue entender las estructuras complejas y las estructuras simplécticas como un mismo tipo de estructuras geométricas generalizadas.

En esta charla se mostrarán varias estructuras geométricas generalizadas que se pueden inducir a partir de estructuras definidas sobre la variedad, utilizando definiciones menos restrictiva que las dadas originalmente por M. Gualtieri. De esta forma, se mostrarán claramente tanto las semejanzas como las diferencias existentes al trabajar sobre el fibrado tangente generalizado en lugar del fibrado tangente o el cotangente.

Agradecimientos Este trabajo está parcialmente financiado por una beca predoctoral "Concepción Arenal" (Universidad de Cantabria y Gobierno de Cantabria).

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VI Congreso de Jóvenes Investigadores

Real Sociedad Matemática Española

León, Febrero de 2023

Futaki invariants for the Hull-Strominger system

R. Gonzalez Molina*

In a highly influential article [1], Reid put forward a proposal for understanding the moduli space of projective Calabi-Yau threefolds. Roughly, components corresponding to different topologies are glued together via complex surgeries on the manifold, called conifold transitions, so that the resulting moduli is rendered irreducible. Interestingly, this process incorporates non-Kähler backgrounds as 'boundary phenomena' on moduli.

Some years later, Yau proposed that the geometrization of conifold transitions should make use of the Hull-Strominger system, and proposed a conjecture for the existence problem in [4]. This system has its origins in theoretical physics as consistency conditions for heterotic supergravity theories, and is now regarded in hermitian geometry as providing canonical geometry in the non-Kähler realm.

In this talk, we will use methods in generalized geometry as iniciated by Hitchin [3] and Gualtieri [2] to recast the equations of the Hull-Strominger system in terms of a Hermite-Einstein condition on a holomorphic Courant algebroid. Consequently, we will derive a family of subtle holomorphic invariants obstructing the existence of solutions beyond the existence of balanced metrics and stability of bundles. These potentially have the power to decide on Yau's conjecture on the existence problem.

Time permitting, we will discuss a conjectural picture for stability conditions for Courant algebroids which are reminiscent of those in GIT, which makes use of a notion of harmonic metrics. This will be illustrated in a family of examples over homogeneous manifolds.

This is joint work with M. Garcia-Fernandez.

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VI Congreso de Jóvenes Investigadores
Real Sociedad Matemática Española
León, Febrero de 2023

Quantizability of manifold invariants

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Topological Quantum Field Theories (TQFTs) are powerful categorical tools for computing algebraic invariants of closed smooth manifolds. From the astonishing works of Witten to quantize Jones polynomials of knots, passing through the reinterpretation of the Donaldson invariants in this context, and reaching the modern developments of extended TQFT via ∞-categories, quantum topology is currently one of the most active research areas.

These exciting results open an intriguing question: Given an algebraic invariant associated to closed manifolds, does there exist a TQFT computing it? Even though in the general case the answer remains unknown, in this talk we will discuss some partial results in this direction, providing necessary and sufficient conditions of quantizability in terms of almost-TQFTs and almost-Frobenius algebras for TQFTs with a geometric meaning. Time permitting, we will exhibit some relevant consequences, as the impossibility of constructing monoidal TQFTs for computing the Poincaré polynomial of moduli spaces of flat connections.

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VI Congreso de Jóvenes Investigadores
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León, Febrero de 2023

Superficies de Bryant completas

Jorge Hidalgo Calderón *

Se llama *superficie de Bryant* a una superficie inmersa de curvatura media constante igual a uno en el espacio hiperbólico de dimensión 3, $\mathbb{H}^3$.

Demostramos un teorema de interpolación muy general para superficies de Bryant completas. En particular, se obtiene que toda superficie de Riemann abierta es la estructura compleja de una superficie de Bryant completa. Además, dicha superficie puede tomarse densa en $\mathbb{H}^3$.

Por la naturaleza del método usado, el resultado sobre superficies de Bryant es un corolario de un resultado más general de aproximación e interpolación tipo Runge-Mergelyan, Carleman, Weierstrass-Florack ([1, Capítulos 1 y 3]) para curvas holomorfas nulas en $\mathbb{C}^2 \times \mathbb{C}^*$, que también tiene consecuencias sobre curvas holomorfas nulas en $SL(2, \mathbb{C})$ y superficies espaciales de curvatura media constante 1 con singularidades admisibles en el espacio de De Sitter $\mathbb{S}^3_1$.

En esta charla proporcionaré una visión general de los resultados conseguidos y explicaré de manera breve algunas de las ideas clave de las demostraciones, así como algunas de sus consecuencias más relevantes.

Esto es un trabajo conjunto con A. Alarcón y I. Castro-Infantes.

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VI Congreso de Jóvenes Investigadores
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León, Febrero de 2023

El problema de Zermelo y su aplicación a la modelización de incendios forestales

Enrique Pendás Recondo*

El problema de navegación de Zermelo consiste en determinar la trayectoria más rápida entre dos puntos para un objeto que se mueve en presencia de una corriente, como un aeroplano en el aire sujeto a la acción del viento. En el caso más general posible en el que permitimos que la velocidad del móvil varíe en el espacio, en el tiempo y en la dirección, la solución a este problema, cuando la interpretamos geométricamente, resulta ser una geodésica luminosa de un cierto espacio-tiempo de Finsler [1].

Cuando consideramos ya no un móvil, sino una onda (en general, anisótropa) que se propaga en todas las direcciones, el problema de Zermelo se transforma en el principio de Fermat: las trayectorias que minimizan el tiempo de propagación y, por tanto, forman el frente de ondas, vuelven a ser geodésicas luminosas de un espacio-tiempo de Finsler [2, 3]. Incluso se puede considerar el caso en el que la velocidad de la onda presenta una discontinuidad, por ejemplo cuando pasa de un medio a otro. En este caso, la trayectoria se refracta siguiendo una ley de Snell generalizada [4].

Todo este marco teórico se puede aplicar no solo para modelizar la propagación de ondas, sino de cualquier fenómeno físico que se comporte como tal (esencialmente, que cumpla el principio de Huygens). En particular, la propagación de incendios forestales constituye una de las aplicaciones más sorprendentes e interesantes, ya que su modelización a través de la geometría de Finsler proporciona ciertas ventajas: por un lado, permite ajustar de forma precisa la anisotropía natural de un incendio (debido, por ejemplo, al viento o la pendiente); y por otro lado, reduce el cálculo del frente del fuego a la resolución de un sistema de ecuaciones diferenciales ordinarias (las ecuaciones de las geodésicas) [5].

Agradecimientos Esta investigación es parte del proyecto PID2021-124157NB-I00, financiado por MCIN/ AEI/ 10.13039/501100011033/ y por FEDER "Una manera de hacer Europa".

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VI Congreso de Jóvenes Investigadores

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León, Febrero de 2023

Solitones de Ricci homogéneos y grupos nilpotentes de Iwasawa

V. Sanmartín-López*

Esta charla se sitúa en la intersección existente entre la geometría de subvariedades de espacios simétricos de tipo no compacto y el estudio de métricas homogéneas Einstein o de solitón de Ricci. Veremos que una de las consecuencias de la recientemente resuelta conjetura de Alekseevskii [1] es que el estudio de los solitones de Ricci homogéneos expanding, en particular de las variedades homogéneas Einstein con curvatura escalar negativa, pasa por la investigación de nilsolitones [3]. Estos últimos pueden a su vez entenderse como cierto tipo de subgrupos de Lie de la parte nilpotente de la descomposición de Iwasawa asociada a los espacios simétricos de tipo no compacto [2], considerados con la métrica inducida. Finalmente y utilizando este punto de vista, presentaremos ejemplos de nilsolitones como subgrupos de Lie de grupos de Iwasawa así como resultados parciales de clasificación de los mismos junto con algunas de sus consecuencias.

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León, Febrero de 2023

Técnicas de análisis geométrico para un problema variacional en geometría (pseudo-)Finsler

Fidel F. Villaseñor*

Comenzaremos motivando el problema de extender la relatividad general mediante la geometría de (Lorentz-)Finsler, desde los puntos de vista tanto físico como puramente matemático. La búsqueda de análogos a la ecuación de campo de Einstein conduce a un funcional natural cuya variación con respecto a una métrica pseudo-Finsler L ha sido estudiada anteriormente [2]. Aquí lo variaremos con respecto a L y una conexión no-lineal independiente N, extendiendo el *formalismo de Palatini* [3]. Tras reducir las ecuaciones obtenidas al caso en que N es libre de torsión, mostraremos cómo (bajo una hipótesis de globalidad en cada T_pM) herramientas variadas de análisis geométrico permiten establecer propiedades de sus soluciones:

(i) En signatura indefinida, un argumento recursivo muestra que N es única si se le exige analiticidad en cada T_pM. Sus geodésicas luminosas coinciden con las de L.

(ii) En signatura lorentziana, el principio clásico del máximo implica que N debe ser Ricci-llana. Si cierto invariante tensorial de L se anula, puede eliminarse la condición de analiticidad de (i).

(iii) Para métricas riemannianas, el conocimiento de los valores propios del laplaciano de la esfera estándar implica que N debe ser la conexión de Levi-Civita.

Esta contribución está basada en el trabajo conjunto con Miguel Ángel Javaloyes y Miguel Sánchez Caja [1].

Agradecimientos FFV ha sido sufragado por la ayuda FPU19/01009 del Ministerio de Universidades, el proyecto PID2020-116126GB-I00 financiado por MCIN/AEI/10.13039/501100011033, el projecto PY20-01391 (PAIDI 2020) financiado por Junta de Andalucía-FEDER, y la unidad de excelencia (IMAG) María de Maeztu CEX2020-001105-M financiada por MCIN/AEI/10.13039/5011000

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Overdetermined elliptic problems in the sphere

J. Wu *

A classical result by J. Serrin [5] states that the existence of a positive solution to the overdetermined elliptic problem

$$\begin{cases} \Delta u + f(u) = 0 & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \\ \partial_\nu u = \text{constant} & \text{on } \partial\Omega, \end{cases}$$

in a bounded domain $\Omega \in \mathbb{R}^d$ implies that Ω must be a ball. The framework of the space form manifolds has been treated in 1998 by S. Kumaresan and J. Prajapat [3]: if Ω is contained in a hemisphere and the problem admits a solution, then Ω must be a geodesic ball. There are examples of nontrivial extremal domains containing a geodesic, see [2]. A question arises naturally: is it true that if $\Omega \subset \mathbb{S}^d$ is simply connected and the problem can be solved, then Ω must be a geodesic ball? In [1], J.M. Espinar and L. Mazet give an affirmative answer to this question if $d = 2$ but under some extra assumptions on the nonlinear term.

In this talk, we will give a negative answer to the above question: there exist nontrivial simply connected domains of the sphere $\mathbb{S}^d$ such that the overdetermined elliptic problem admits a positive solution. This shows in particular that Serrin's theorem for overdetermined problems in the Euclidean space cannot be generalized to the sphere even for simply connected domains. This is a joint work with D. Ruiz and P. Sicbaldi, see [4].

Agradecimientos J. W. has been supported by the China Scholarship Council (CSC201906290013) and by J. Andalucia (FQM-116).

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Differential Geometry, Mathematical Physics and Control Theory

Organizadores: Miguel Ángel Berbel, Asier López y Xavier Ribas

Jueves 9 de Febrero
Moderadores: *Asier López Gordón, Xavier Rivas Guijarro y Miguel Ángel Berbel López*

10:00 *Shannon information entropy for the Darboux III quantum nonlinear oscillator*, Iván Gutiérrez Sagredo

10:30 *Periodic orbits and Birkhoff sections of stable Hamiltonian structures*, Robert Cardona

11:30 *Reduction by symmetries and applications to optimal control*, Leonardo J. Colombo

12:15 *Nonholonomic systems and the inverse problem*, Marta Farré Puiggalí

12:45 *An introduction to contact Hamiltonian systems*, Manuel Lainz

17:00 *Noether Symmetries in Classical Field Theory and Pre-multisymplectic Geometry*, Arnoldo Guerra IV

17:30 *Contracting Forced Lagrangian and Contact Lagrangian Systems: application to nonholonomic systems with symmetries*, Alexandre Anahory Simoes

18:00 *Variational integrators for reduced Lagrange–Dirac systems*, Álvaro Rodríguez Abella

18:30 *A polycosymplectic reduction*, Bartosz Maciej Zawora

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Contracting Forced Lagrangian and Contact Lagrangian Systems: application to nonholonomic systems with symmetries

Alexandre Anahory Simoes [*], Leonardo Colombo [†]

Contraction theory is one of the analytical tools used in nonlinear analysis to study the stability of dynamical systems (see [1]). Loosely speaking, if a system exhibits contracting behavior then there exists a unique fixed point of the dynamics towards which all dynamical solutions converge exponentially fast.

We begin the talk with a review of contraction theory in general Riemannian manifolds, as introduced in [2], with some standard examples appearing in the literature

Then, we will address the problem of identifying contracting systems among dynamical systems appearing in mechanics. To this end, we introduce first a new strategy allowing to identify contracting systems in a general Riemannian manifold. Using this technique, we prove that a family of dissipative forced mechanical systems is contracting, while stating immediate consequences of this fact for the stability of these systems. Finally, we use the previous results to study the stability of families of Contact and Nonholonomic systems.

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Periodic orbits and Birkhoff sections of stable Hamiltonian structures

Robert Cardona*

In this talk, we are interested in the dynamics of non-vanishing volume-preserving vector fields on closed three-manifolds. We will survey some recent striking results for the class of Reeb vector fields defined by contact forms, and present some generalizations to the case of Reeb fields defined by stable Hamiltonian structures. Such structures appear, for example, on a Hamiltonian system restricted to a "stable" energy level set, or on stationary solutions to the equations that model ideal fluids. The kind of results that we will discuss involve the existence of periodic orbits and the existence of a Birkhoff section. The existence of such a section reduces the study of the vector field to a two-dimensional problem. This talk is based on joint work with A. Rechtman.

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Reduction by symmetries and applications to optimal control

Leonardo J. Colombo*

The most celebrated technique in geometric mechanics is the reduction by symmetries of mechanical systems. In this introductory talk I will make a review of the different reduction by symmetries techniques in the geometric mechanics literature and I will show applications of the use of this technique to optimal control problems of mechanical systems.

Agradecimientos L.C acknowledge financial support from Grant PID2019-106715GB-C21 funded by MCIN/AEI/ 10.13039/501100011033.

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Nonholonomic systems and the inverse problem

M. Farré Puiggalí *

The inverse problem addresses the question of whether or not a system of second order ordinary differential equations is equivalent to a regular Lagrangian system. If this is the case then we say that the system is variational. EDS (exterior differential systems) techniques have been used to identify variational cases. I will present a notion of variationality for constrained systems, in particular nonholonomic systems, and discuss the possible application of EDS techniques in this constrained case.

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Noether Symmetries in Classical Field Theory and Pre-multisymplectic Geometry

Arnoldo Guerra IV*

A general overview of Noether symmetries in classical field theories will be given followed by a general discussion on constraints arising in singular field theories in the pre-multisymplectic framework. The behavior of symmetries in the presence of such constraints will be the main topic. Various field theories, including Carrollian scalar field theories, Chern-Simons theory, and non-Abelian Yang–Mills, will be used as examples to illustrate the features of Noether symmetries exhibited by singular field theories.

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Shannon information entropy for the Darboux III quantum nonlinear oscillator

I. Gutierrez-Sagredo*

In this talk I will present some recent results [1] concerning the Shannon information entropy [2] for the quantum version of the Darboux III oscillator. The so-called Darboux III oscillator [3] is an exactly solvable N-dimensional nonlinear oscillator defined on a radially symmetric space with non-constant negative curvature. This oscillator can be interpreted as a smooth (super)integrable deformation of the usual N-dimensional harmonic oscillator in terms of a non-negative parameter λ which is directly related to the curvature of the underlying space.

In particular, analytical results for the Shannon entropy in the position space can be found in the N-dimensional case, and the known results for the quantum states of the N-dimensional harmonic oscillator are recovered in the limit of vanishing curvature $\lambda \to 0$. However, the Fourier transform of the Darboux III wave functions cannot be computed in exact form, thus preventing the analytical study of the information entropy in momentum space.

Nevertheless, the latter can be computed numerically and we find that by increasing the absolute value of the negative curvature (through a larger λ parameter) the information entropy in position space increases, while in momentum space it becomes smaller. This result is indeed consistent with the spreading properties of the wave functions of this quantum nonlinear oscillator, which will be explicitly shown.

Agradecimientos This work has been partially supported by Agencia Estatal de Investigación (Spain) under grant PID2019-106802GB-I00/AEI/10.13039/501100011033, by the Regional Government of Castilla y León (Junta de Castilla y León, Spain) and by the Spanish Ministry of Science and Innovation MICIN and the European Union NextGenerationEU/PRTR, as well as the contribution of the European Cooperation in Science and Technology through the COST Action CA18108.

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An introduction to contact Hamiltonian systems

Manuel Lainz *

Contact Hamiltonian systems are a generalization of the Hamiltonian systems of classical mechanics. The action is added as an extra variable in phase space, and symplectic geometry is changed by contact geometry. In this way, we are able to model a large new class of systems. Indeed, symplectic geometry is unable to deal with dynamics where there is energy dissipation. Nonetheless, this is possible in the contact world. In the recent years, there is a growing interest in the applications of contact Hamiltonian systems, including thermodynamics and mechanical systems with dissipation.

In this talk we will introduce the Hamiltonian and Lagrangian formalism, including the variational formulation through the Herglotz principle. We will explain the basic results regarding symmetries, dynamics and geometric properties of this systems. We will also discuss some of the new applications.

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Variational integrators for reduced Lagrange–Dirac systems

Á. Rodríguez Abella*   M. Leok†

Lagrange–Dirac mechanics provides a unifying framework for treating degenerate systems with nonholonomic constraints, and may be formulated either by means of Dirac structures [1] or by means of the Hamilton–Pontryagin variational principle [2]. In addition, Lagrange–Poincaré reduction has been extended to this context [3]. The corresponding reduced equations are obtained from the so-called gauged Dirac structure on the reduced space, which is induced from the canonical Dirac structure on the cotangent bundle of the configuration manifold.

On the other hand, a discrete (in time) notion of Dirac mechanics has been formulated by using a discrete analogue of the Tulczyjew triple [4]. This allows for building symplectic variational integrators for degenerate systems with holonomic constraints.

This aim of this talk, which is based in our recent work [5], is to present the Lagrange–Poincaré reduction theory for discrete Lagrange–Dirac systems. We restrict ourselves to unconstrained systems and abelian symmetry groups. The reduction procedure relies on the notion of discrete principal connections on principal bundles [6] and is performed by reducing both the discrete Dirac structure and the discrete Hamilton-–Pontryagin principle. Both approaches are shown to give the same discrete reduced equations. To conclude, the double spherical pendulum is examined in order to illustrate the use of our theory in the construction of variational integrators for the reduced systems.

Agradecimientos ARA was supported by the Ministerio de Universidades (Spain) under an FPU grant, and partially supported by Ministerio de Ciencia e Innovación (Spain), under grants PGC2018-098321-B-I00 and PID2021-126124NB-I00. ML was partially supported by the NSF under grants DMS-1411792, DMS-1345013, DMS-1813635, CCF-2112665, by AFOSR under grant FA9550-18-1-0288, and by the DoD under grant HQ00342010023 (Newton Award for Transformative Ideas during the COVID-19 Pandemic).

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A polycosymplectic reduction

Bartosz Maciej Zawora*

A polysymplectic manifold is $(k+1)$-tuple $(P, \omega^1, \ldots, \omega^k)$, where P is a manifold, while $\omega^\alpha \in \Omega^2(P)$, with $\alpha = 1, \ldots, k$, are closed differential forms such that $\bigcap_{\alpha=1}^k \ker \omega^\alpha = \{0\}$. The polysymplectic Marsden-Weinstein reduction theorem reduces a class of polysymplectic Hamiltonian systems, let us say $(P, \omega^1, \ldots, \omega^k, h)$, admitting a Lie group action $\Phi : G \times P \to P$ leaving $h \in C^\infty(P)$ and the $\omega^1, \ldots, \omega^k$ invariant. In particular, via the so-called Ad^k-equivariant polysymplectic momentum map $\mathbf{J}^\Phi : P \to (\mathfrak{g}^*)^k$ induced by Φ, where $\mathfrak{g}$ is the Lie algebra of G, one can find conditions to guarantee the existence of a reduced polysymplectic Hamiltonian system $(\mathbf{J}^{\Phi-1}(\mu)/G_\mu, \omega^1_\mu, \ldots, \omega^k_\mu, h_\mu)$, where G_μ is the isotropy subgroup of $\mu \in (\mathfrak{g}^*)^k$ relative to the coadjoint action $\mathrm{Ad}^k : G \times (\mathfrak{g}^*)^k \to (\mathfrak{g}^*)^k$ and h_μ is a function on $\mathbf{J}^{\Phi-1}(\mu)/G_\mu$ whose pull-back to $\mathbf{J}^{\phi-1}(\mu)$ retrieves the value of h on it.

My talk presents the Marsden-Weinstein reduction theorem for polycosymplectic manifolds $(M, \omega^1, \ldots, \omega^k, \tau^1, \ldots, \tau^k)$, where M is a manifold, while $\omega^\alpha \in \Omega^2(M)$ and $\tau^\alpha \in \Omega^1(M)$, for every $\alpha = 1, \ldots, k$, are closed differential forms such that $\bigcap_{\alpha=1}^k (\ker \omega^\alpha \cap \ker \tau^\alpha) = \{0\}$, with a Lie group action $\Psi : G \times M \to M$ leaving ω^α and τ^α invariant for every $\alpha = 1, \ldots, k$. First, I will show how a polycosymplectic manifold $(M, \omega^1, \ldots, \omega^k, \tau^1, \ldots, \tau^k)$ with an Ad^k-equivariant polycosymplectic momentum map $\mathbf{J}^\Psi$ extends to a polysymplectic manifold with polysymplectic momentum map. Then, I will present how the polycosymplectic reduction can be reformulated by using this extension, and how one can obtain the reduced polycosymplectic Hamiltonian system $(\mathbf{J}^{\Psi-1}(\mu)/G_\mu, \omega^1_\mu, \ldots, \omega^k_\mu, \tau^1_\mu, \ldots, \tau^k_\mu, h_\mu)$ from the Marsden-Weinstein reduction theorem for polysymplectic manifolds. Moreover, I will discuss the Ad^k-equiv
-ariance condition for polysymplectic and polycosymplectic momentum maps, and I will demonstrate how these reductions generalise to non-Ad^k-equivariant momentum maps.

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Teoría de Grupos

Organizadores: María Cumplido y Iván Chércoles

Jueves 9 de Febrero
Moderadores: *María Cumplido e Iván Chércoles*

17:00 *A Kurosh-type theorem for Lie algebras*, Simone Blumer

17:20 *Cohomological uniqueness of finite groups*, Lander Guerrero

17:40 *Hypercubical groups: a trip from RAAGs to Borromean rings*, Alberto Cassella

18:00 *Abelian splittings of pro-p Right-angled Artin groups*, Matteo Pintonello

18:20 *Rigidez finita de complejos de curvas no separantes*, Rodrigo de Pool

Viernes 10 de Febrero
Moderadores: *María Cumplido e Iván Chércoles*

10:00 *Hausdorff dimension of groups acting on trees*, Marialaura Noce

10:30 *Structure of Finitely Presented Bestvina-Brady Groups*, Mallika Roy

11:30 *Cohomology and fusion systems*, Oihana Garayalde

12:00 *Core-portraits in self-similar groups*, Jone Uria

12:30 *Automorphism groups of Cayley graphs of Coxeter groups*, Federico Berlai

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A Kurosh-type theorem for Lie algebras

Simone Blumer*

Kurosh theorem for groups provides the structure of any subgroup of a free product of groups and one of its proofs relies on Bass-Serre theory of groups acting on trees. In the case of Lie algebras, such an elegant theory does not exist and the analogue of Kurosh theorem fails in general, as it was first noticed by A.I. Shirshov [3]. However, in [1], we prove that, for a class of positively graded Lie algebras satisfying certain local properties in cohomology, an analogue of such a structure theorem holds true for subalgebras generated by elements of degree 1. Such class consists of Koszul Lie algebras, in which all the subalgebras that are generated in degree 1 are Koszul.

The talk has the aim of providing an introduction to the theory of graded Lie algebras, with a focus on their cohomological theory, which allows one to recognise some features of their own minimal presentations.

Acknowledgment The speaker would like to thank the University of Zaragoza for giving him the opportunity of spending one year of research abroad and for partially supporting him. He also wishes to thank Prof. C. Martínez-Pérez and Prof. T. Weigel for very helpful conversations on (co)homological arguments which have been used in the proofs.

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Hypercubical groups: a trip from RAAGs to Borromean rings

A. Cassella*

A hypercubical group is a group G for which a certain specific cubical complex constructed starting from the Cayley graph of G with respect to a generating set S is contractible. Such cubical complex can be seen as the cubical version of a flag complex with given 1-skeleton. The definition of hypercubical group was inspired by what happens in the case of right-angled Artin groups (RAAGs for short): the above-mentioned cubical complex is a cubical model for the universal covering of the Salvetti complex, thus being contractible. The aim of this talk is to introduce the family of hypercubical groups and some geometric notions related to them, and to show some interesting examples that go from some generalizations of RAAGs to some knot-theoretic groups and more, showing how this kind of construction can be of great interest.

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Automorphism groups of Cayley graphs of Coxeter groups

A. Federico Berlai*

The automorphism group of a locally finite graph naturally admits a locally compact totally disconnected topology where basic open sets are vertex stabilizers. Thus, any such group is a tdlc group. It could be discrete (this happens exactly when the group is countable) or have a non trivial topology. In this talk I will focus on a particular family of locally finite graphs, that is Cayley graphs of Coxeter groups with respect to a standard generating set. I will characterize exactly when the topology of these automorphism groups is not the discrete topology (that is, when it is of interest) in terms of properties of a finite graph defining the Coxeter group.

Joint work with Michal Ferov.

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León, Febrero de 2023

Cohomology and fusion systems

O. Garaialde Ocaña*

Given a finite group G and a prime number p that divides the size of G, on the one hand, one can study how the p-subgroups of G are fused under conjugation by the elements of G. This is accomplished using the theory of *fusion systems*.

On the other hand, one can study the cohomology ring $H^*(G;\mathbb{F}_p)$ of G with trivial coefficients in the finite field $\mathbb{F}_p$ of p elements. As it is common in mathematics, we seek for ‘significant’ algebraic properties of G to be encoded in the ring $H^*(G;\mathbb{F}_p)$. But one of the major difficulties in this topic is that such rings are very hard to compute. Even more, it is difficult to foresee whether two given groups will have isomorphic cohomology rings or not. If the given group G has a normal subgroup, one of the main tools to compute its cohomology ring is the Lyndon-Hochschild-Serre spectral sequence.

A priori, the theory of fusion systems of finite groups and applying spectral sequences to compute $H^*(G;\mathbb{F}_p)$ may seem to be unrelated research areas. The aim of this talk is to employ fusion systems to describe a new spectral sequence that allows to compute $H^*(G;\mathbb{F}_p)$ even when G is simple (with a normal subgroup in a Sylow p-subgroup) and to provide an example of such computation. No prior knowledge of the aforementioned subjects is required to follow the talk.

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Cohomological uniqueness of finite groups

Lander Guerrero Sánchez*

Given a finite group G, we can define its cohomology $\mathrm{H}^\bullet(G, K)$ with coefficients on a field K, which has the structure of a graded-commutative algebra over K. It is then natural to ask how well the structure of G is encoded in its cohomology algebra, and whether the cohomology is unique when restricting to certain families of groups.

In this talk, we will study the mod-p cohomologies of some families of finite p-groups in order to determine how similar they are.

This is based on joint work with Oihana Garaialde Ocaña and Jon González-Sánchez.

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Hausdorff dimension of groups acting on trees

Marialaura Noce*

The concept of Hausdorff dimension was defined in the 1930s and is a way of measuring the relative size of a subset in a metric space. In the 1990s Abercrombie, Barnea and Shalev have considered the Hausdorff dimension in the setting of (countably based) profinite groups. In this talk, we will survey known results concerning the Hausdorff dimension of (the topological closure of) some distinguished families of subgroups of groups acting on trees, and we present new results and open research directions about this topic.

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Abelian splittings of pro-p Right-angled Artin groups

Matteo Pintonello*

Given a finite abstract graph Γ, the associated Right-angled Artin group (usually abbreviated as RAAG) is the group generated by the vertices of Γ defined by the presentation

$$G_\Gamma = \langle V(\Gamma) | [u, v] = 1 \text{ if and only if } u \text{ and } v \text{ are adjacent in } \Gamma \rangle.$$

Pro-p RAAGs are defined as the pro-p completion of abstract RAAGs.
In 2015 Groves and Hull characterized how an abstract RAAG can split as an amalgamated product or HNN extension over an abelian subgroup. Using the pro-p version of Bass-Serre Theory developed by Mel'nikov, Ribes, Zalesski et al. we will obtain an analogous description for splittings of pro-p RAAGs over abelian pro-p subgroups.
This is a joint work with M. Casals Ruiz and P. Zalesski.

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VI Congreso de Jóvenes Investigadores

Real Sociedad Matemática Española

León, Febrero de 2023

Rigidez finita de complejos de curvas no separantes.

Rodrigo De Pool*

El grupo modular $\mathrm{Mod}(S)$ es el grupo de homeomorfismos de la superficie S, salvo homotopía. El estudio de este grupo resulta huidizo y, a menudo, se estudia mediante su acción en determinados espacios simpliciales. En particular, $\mathrm{Mod}(S)$ actúa de forma natural sobre el conjunto de curvas en la superficie S. En esta charla veremos cómo se le puede dotar de una estructura simplicial a este conjunto, de modo que la acción de $\mathrm{Mod}(S)$ se extienda a todo el complejo simplicial. Más aún, introduciremos propiedades de estos espacios como la *rigidez finita* y las consecuencias que tiene sobre la estructura del grupo modular.

Agradecimientos A Javier Aramayona por presentarme el problema y guiarme en su resolución. A los organizadores de la sesión paralela de teoría de grupos, porque gracias a ellos tenemos este espacio. A Iván Chércoles por animarme a dar esta charla.
El autor reconoce apoyo financiero de la ayuda CEX2019-000904-S financiada por MCIN/AEI/ 10.13039/501100011033.

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VI Congreso de Jóvenes Investigadores
Real Sociedad Matemática Española
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Structure of Finitely Presented Bestvina-Brady Groups

Mallika Roy*

Right-angled Artin groups (RAAGs, for short) and their subgroups are of great interest because of their geometric, combinatorial and algorithmic properties. It is convenient to define these groups using finite simplicial graphs. The isomorphism type of the group is uniquely determined by the graph. Moreover, many structural properties of right angled Artin groups can be expressed in terms of their defining graph.

In this talk, I address the question of understanding the structure of a class of subgroups of RAAGs in terms of the graph. Bestvina and Brady, in their seminal work, studied these subgroups (now called Bestvina-Brady groups or Artin kernels) from finiteness conditions viewpoint. A Bestvina–Brady group is the kernel of the group homomorphism from a RAAG to $\mathbb{Z}$ sending all generators of the RAAG to 1. Unlike the RAAGs the isomorphism type of Bestvina-Brady groups is not uniquely determined by the defining graph. We prove that certain finitely presented Bestvina-Brady groups can be expressed as an iterated amalgamated product of RAAGs. Moreover, we show that this amalgamated product can be read off from the graph defining the ambient RAAG.

In this talk, I also discuss the case where all the generators of RAAG are not mapped to 1. I give the presentation of the kernal of a map which maps the generators of a RAAG to some integer.

This is partly a joint work with *Montserrat Casals* and partly a joint work with *Priyavrat Deshpande*.

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Core-portraits in self-similar groups

J. Uria-Albizuri*

Self-similar groups have been widely studied since 1980's, when the first Grigorchuk group was defined. The importance of such groups resides in the amount of important (counter)examples provided for solving important group theoretical problems, such as the General Burnside Problem or the Milnor Problem, [Nek05].

Self-similar groups can be seen as groups acting on a regular rooted tree. An automorphism of a d-adic rooted tree T can be described by decorating the tree vertices with permutations from the symmetric group S_d. However, we can also decorate the leaves by tree automorphisms, providing in this way a finite description of an automorphism.

On the other hand, if the action of a group G on T is contracting, there is a finite subset of the group G, called the nucleus, for which every element in G admits such a finite description with all the leaves decorated by elements in the nucleus. The minimal such description for an element is called its core-portrait [Gri05].

We provide the first explicit description of the core-portraits of a self-similar group. Namely, we give such a despription for every non-symmetric Grigorchuk-Gupta-Sidki group.

Agradecimientos The author aknowledges support from the Spanish Government, grants PID2020-117281GB-I00 and PID2019-107444GA-I00, partly from European Regional Development Fund (ERDF), and the Basque Government, grant IT1483-22.

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Álgebras No Asociativas

Organizadores: Alejandra Sarina Córdova, Xavier García y María Pilar Páez

Martes 7 de Febrero
Moderadora: *María Pilar Páez Guillán*

11:30 *Nilpotent Poisson-type algebras*, Amir Fernández Ouaridi

12:00 *Ideales invariantes en álgebras de caminos de Leavitt*, Cristóbal Gil Canto

12:30 *Dimensión de Gelfand-Kirillov y entropía de álgebras filtradas*, Iván Ruiz Campos

13:00 *Gradings induced by nilpotent elements*, Guillermo Vera de Salas

Miércoles 8 de Febrero
Moderadores: *Alejandra Sarina Córdova Martínez y Xabier García Martínez*

10:00 *Natural families in evolution algebras*, Yolanda Cabrera Casado

10:30 *Grupos de automorfismos de las álgebras de evolución de Cayley*, Alicia Tocino Sánchez

11:30 *On metric generalized Jordan superpairs*, Diego Aranda Orna

12:00 *Categorías tensoriales, álgebras y superálgebras*, Alberto Daza García

12:30 *Local quadratic Lie algebras*, Jorge Roldán López

13:00 *Divisibilidad en álgebras Hom-asociativas*, Germán García Butenegro

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Real Sociedad Matemática Española

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On metric generalized Jordan superpairs

D. Aranda-Orna*

A generalization of the Faulkner construction is given for generalized Jordan superpairs, and some operators (e.g., tensor superproducts) are transferred from the class of metric Lie supermodules (Lie supermodules with a nondegenerate even supersymmetric invariant bilinear form on the Lie superalgebra) to the class of metric generalized Jordan superpairs (generalized Jordan superpairs with a nondegenerate even supersymmetric invariant bilinear form). We only consider finite-dimensional objects and over a field of characteristic different from 2.

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Natural families in evolution algebras

Yolanda Cabrera Casado*

The modeling of non-mendelian genetics brought forth a new type of genetic algebras called evolution algebras. Basic concepts of evolution algebras of arbitrary dimension are studied. The notion of the range of evolution, natural vector, and subspace of evolution is introduced, and there is a decomposition of evolution algebras relative to the latter. This is a joint work with Mercedes Siles Molina and Nadia Boudi.

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VI Congreso de Jóvenes Investigadores

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Categorías tensoriales, álgebras y superálgebras

A. Daza-García,*

En esta charla nos centraremos en tres categorías tensoriales sobre un cuerpo de característica 3: la categoría $\mathrm{Rep}C_3$ de representaciones de dimensión finita del grupo cíclico de orden 3, la categoría Verñinde Ver_3 y la categoría sVec de super espacios vectoriales de dimensión finita. En cada una de estas categorías podemos definir el concepto de álgebra. Por ejemplo, en la categoría $\mathrm{Rep}C_3$, un álgebra es equivalente a un álgebra ordinaria junto con un automorfismo de orden 3, mientras que en sVec, un álgebra es una superálgebra.

En característica 3, la categoría $\mathrm{Rep}C_3$ no es semisimple, pero podemos usar el concepto de semisimplificación para definir la categoría semisimple Ver_3 junto con un funtor $S\colon \mathrm{Rep}C_3 \to \mathrm{Ver}_3$ llamado el funtor de semisimplificación. Además, Ver_3 es una categoría equivalente a la categoría de superespacios vectoriales. En esta charla, explicaremos cómo podemos usar esto para construir superálgebras partiendo de álgebras en caracteríastica 3 y lo aplicaremos al álgebra de Cayley Split con automorfismos de orden 3.

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Nilpotent Poisson-type algebras*

Amir Fernández Ouaridi†

A *Poisson-type algebra* $(\mathcal{P}, \cdot, \{\cdot, \cdot\})$ is a vector space $\mathcal{P}$ equipped with a commutative associative multiplication $\cdot$ and another multiplication $\{\cdot, \cdot\}$ satisfying some family of polynomial identities. These two multiplications are also required to satisfy a family of Leibniz type identities involving both multiplications. The main example of Poisson-type algebras are Poisson algebras; other examples include Leibniz-Poisson algebras, transposed Poisson algebras, Novikov-Poisson algebras or commutative pre-Lie algebras.

The first historical encounter with Poisson-type algebras is in 1809 with Hamiltonian mechanics, where the algebra of smooth functions on variables the space coordinates q_i and the corresponding momenta p_i is endowed with a Lie bracket which satisfies a Leibniz compatibility rule, namely, the Lie bracket is a derivation. This structure, the Hamiltonian and the Hamilton equations make a system that is widely used to describe any kind of environment in classical mechanics and that provides a solid theoretical foundation for the later generalization to quantum mechanics. With the abstraction of many notions in Physics, Hamiltonian systems were geometrized into manifolds that model the set of all possible configurations of the system, and the cotangent bundle of this manifold describes its phase space, which is endowed with a Poisson algebra. Later, Poisson algebras led to other algebraic structures, and the notion of Poisson-type algebra arose.

A Poisson-type algebra $(\mathcal{P}, \cdot, \{-,-\})$ is called *nilpotent* if the sequence of subspaces given by $\mathcal{P}^{(1)} := \mathcal{P}$ and for $n \geq 1$,

$$\mathcal{P}^{(n+1)} := \sum_{i=1}^{n} (\mathcal{P}^{(i)} \cdot \mathcal{P}^{(n+1-i)} + \left\{\mathcal{P}^{(i)}, \mathcal{P}^{(n+1-i)}\right\}),$$

terminates. Given a non-trivial Poisson-type algebra $\mathcal{P}$, it is nilpotent if and only if $\mathcal{P}/\mathrm{Ann}(\mathcal{P})$ is nilpotent, where

$$\mathrm{Ann}(\mathcal{P}) := \{x \in \mathcal{P} : x \cdot \mathcal{P} = \{x, \mathcal{P}\} + \{\mathcal{P}, x\} = 0\}$$

denotes the annihilator of a Poisson-type algebra.

It is well known that every nilpotent algebra is a central extension of a lower dimensional nilpotent algebra. A similar result can be shown when we have two multiplications, as in the case of Poisson-type algebras. The study of the central extensions of non-associative algebras using cohomology has given rise to numerous results (see [2], for example). In this talk, based on results in [1], we will introduce a generalization of the classical cohomological techniques used to obtain the algebraic classification of the central extensions of a given algebra into the context of Poisson-type algebras, and we are going to discuss how we use them to classify low-dimensional nilpotent Poisson algebras.

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Real Sociedad Matemática Española

León, Febrero de 2023

Divisibilidad en álgebras Hom-asociativas

A. Germán García Butenegro*

Resumen. El carácter transitivo de la divisibilidad es una característica fundamental de las álgebras asociativas. En general, esta propiedad requiere que uno o más de los factores sea un elemento central del álgebra.

Se puede encontrar una propiedad similar en álgebras Hom-asociativas. Dada la naturaleza quebrada de las Hom-álgebras, es posible establecer propiedades paralelas en términos de cadenas de divisibilidad y divisores de cero utilizando la aplicación lineal con la que definimos las Hom-álgebras.

Extenderemos estas ideas a diversas clases de álgebras Hom-asociativas.

Abstract. Transitive character of divisibility is a fundamental feature of associative algebras. In general, this property relies on one factor being a central element of the algebra.

A similar feature can be found in Hom-associative algebras. Considering that Hom-algebras are twisted versions of algebras, it is possible to establish analogous properties related to divisibility chains and zero divisors using the twisting linear map.

We extend these ideas to various known and new classes of non-associative Hom-algebras.

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León, Febrero de 2023

Ideales invariantes en álgebras de caminos de Leavitt

Cristóbal Gil Canto [*]

Las álgebras de caminos de Leavitt son una generalización natural de las álgebras de caminos o álgebras asociadas a grafos. Son álgebras que incluyen a aquellas que no satisfacen la propiedad del Número Invariante de la Base, originalmente introducidas por Leavitt. Además, son el análogo algebraico de las C^*-álgebras. Como ejemplos bien conocidos de álgebras de caminos de Leavitt están los llamados colores primarios que se corresponden respectivamente con el ideal generado por el conjunto de vértices en línea, el generado por el conjunto de vértices en ciclos sin salidas y el de vértices en ciclos extremos. Es bien sabido que estos ideales son invariantes por isomorfismos. En esta charla analizaremos la invarianza mediante isomorfismos de otra pieza clave de un álgebra de caminos de Leavitt. Veremos que el ideal generado por los vértices cuyo árbol contiene infinitas bifurcaciones (o al menos un emisor infinito) no es invariante. A pesar de ello encontraremos un sustituto natural que sí satisface la condición de invarianza. Para ello daremos un procedimiento para construir ideales invariantes a partir de otros conocidos que sí lo sean, introduciendo una topología en el conjunto de vértices del grafo y, vía teoría de categorías, pensando en los conjuntos de vértices hereditarios y saturados como funtores. Este es un trabajo conjunto con Dolores Martín Barquero y Cándido Martín González.

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León, Febrero de 2023

Local quadratic Lie algebras

J. Roldán-López*

A local Lie algebra is a Lie algebra with a unique maximal ideal. A quadratic algebra is an algebra that admits an invariant symmetric and non-degenerate bilinear form. The dimension of the vector space of invariant bilinear forms of a Lie algebra is called quadratic dimension of the algebra. Irreducible quadratic Lie algebras of quadratic dimension one are simple, and for dimension two are local algebras. By duality, the orthogonal of a maximal ideal in a quadratic Lie algebra is a minimal ideal, so local quadratic algebras are either solvable with one-dimensional centre or perfect algebras with a simple Levi subalgebra. The converse is not true. In this talk we will focus on the existence and constructions of local quadratic algebras and their quadratic dimensions. This is a joint work with P. Benito.

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VI Congreso de Jóvenes Investigadores

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Dimensión de Gelfand-Kirillov y entropía de álgebras filtradas.

I. Ruiz Campos *

Uno de los invariantes algebraicos que se utilizan a la hora de clasificar álgebras es la dimensión. Esta es condición necesaria para la existencia de isomorfismo. Sin embargo, en cuanto trabajamos con álgebras de dimensión infinita, este invariante es transparente. Es entonces cuando entran en juego nuevos invariantes o nuevas dimensiones como son la dimensión de Gelfand-Kirillov. Este concepto de dimensión se ha utilizado en todo tipo de álgebras: álgebras de caminos [3], álgebras de caminos de Leavitt [4], álgebras de Jordan [2], álgebras de Lie, etc. permitiendo en algunos casos la clasificación de estas. La dimensión de Gelfand-Kirillov surge a partir del concepto de función de crecimiento del álgebra. Cuando esta sea de orden polinómico, la dimensión de Gelfand-Kirillov tendrá valor finito y medirá el grado del orden.

Cuando el crecimiento del álgebra supera el orden polinómico, la dimensión de Gelfand-Kirillov alcanzará valor infinito y nos encontraremos en la misma situación inicial. Tenemos un invariante muy útil, pero no infalible. Para resolver esto, en el caso de grupos y de álgebras filtradas, se define el concepto de entropía [5, 6]. En mecánica estadística la entropía "cuenta "el número de estados accesibles de un sistema. La entropía algebraica actúa de forma similar midiendo el crecimiento de sucesivos espacios vectoriales. En el estudio de la entropía para álgebras filtradas nos va a interesar el caso de álgebras de caminos de grafos finitos pues su cálculo se simplifica considerablemente al trabajar con su matriz de adyacencia.

Dimensión, dimensión de Gelfand-Kirillov y entropía forman un sistema de invariantes de gran interés pues permiten una clasificación más fina de las álgebras filtradas.

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VI Congreso de Jóvenes Investigadores

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León, Febrero de 2023

Grupos de automorfismos de las álgebras de evolución de Cayley

Cristina Costoya* Vicente Muñoz** Alicia Tocino*** Antonio Viruel**

En esta charla se recuerda que las álgebras de evolución sobre cualquier cuerpo son finitamente universales. Esto es, dado cualquier grupo finito G existen infinitas álgebras de evolución perfectas X tales que $\mathrm{Aut}(X) \cong G$. La demostración se basa en la construcción de un functor covariante de una subcategoría de grafos simples finitos (no dirigidos) a la categoría de álgebras de evolución perfectas y finitas.
Además, se construye un nuevo tipo de álgebra de evolución, que llamaremos *álgebra de evolución de Cayley*. Probaremos que estas álgebras son simples. Por último, demostraremos que, si tenemos un cuerpo $\mathbb{K}$ lo suficientemente grande, por ejmplo, si $\mathbb{K}$ es infinito, entonces todo grupo finito G es isomorfo a $\mathrm{Aut}(X)$ donde X es un álgebra de evolución de Cayley finita y simple sobre $\mathbb{K}$.

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VI Congreso de Jóvenes Investigadores
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León, Febrero de 2023

Gradings induced by nilpotent elements

A. E. García [*], B. M. Gómez Lozano [†], C. R. Muñoz Alcázar [†], D. G. Vera de Salas [*]

This talk is about the paper [2].

An element a is nilpotent last-regular if it is nilpotent and its last nonzero power is von Neumann regular. In this paper we show that any nilpotent last-regular element a in an associative algebra R over a ring of scalars Φ gives rise to a complete system of orthogonal idempotents that induces a finite $\mathbf{Z}$-grading on R; we also show that such element gives rise to an $\mathfrak{sl}_2$-triple in R with semisimple adjoint map ad_h, and that the grading of R with respect to the complete system of orthogonal idempotents is a refinement of the Φ-grading induced by the eigespaces of ad_h. These results can be adapted to nilpotent elements a with all their power von Neumann regular, in which case the element a can be completed to an $\mathfrak{sl}_2$-triple and a is homogeneous of degree 2 both in the $\mathbf{Z}$-grading of R and in the Φ-grading given by the eigenspaces of ad_h.

Conversely, when there are enough invertible elements in Φ, we will show that if a nilpotent element a can be completed to an $\mathfrak{sl}_2$-triple, then all powers a^k are von Neumann regular.

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Teoría de Singularidades

Organizadores: Patricio Almirón y Carlos Jesús Moreno

Lunes 6 de Febrero
Moderador: *Patricio Almirón*

18:15 *El Teorema de Comparación Logarítmica y la Euler-homogeneidad en divisores libres*, Abraham Del Valle Rodríguez

18:45 *Zeta functions, abelian orbifold resolutions of singularities and the geometry of curves in weighted projective planes*, Juan Viu Sos

19:15 *Computing a difficult invariant of holomorphic map germs*, Roberto Giménez Conejero

Martes 7 de Febrero
Moderadores: *Abraham Del Valle Rodríguez y Elvira Pérez Callejo*

10:00 *Study of algebraic integrability of foliations on Hirzebruch surfaces. An algorithm of computation depending on the genus*, Elvira Pérez Callejo

10:30 *The Seshadri constant for a divisorial valuation*, Carlos Jesús Moreno Ávila

18:15 *Singularidades simples de foliaciones holomorfas de codimensión uno*, Beatriz Molina-Samper

18:45 *Interpretación geométrica de la base estándar de una curva plana*, David Senovilla Sanz

19:15 *Configuración local de ciclos de campos de vectores analíticos en* $\mathbb{R}^3$, María Martín Vega

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Singularidades simples de foliaciones holomorfas de codimensión uno

Felipe Cano* - Beatriz Molina-Samper *

Las singularidades simples son las singularides más sencillas que se espera obtener tras un proceso de “reducción de singularidades” de una foliación holomorfa de codimensión uno: generalizan, al caso de foliaciones, la situación de que una función sea monomial.

Estudiaremos las soluciones analíticas (hipersuperficies invariantes) alrededor de una singularidad simple y algunas propiedades dinámicas como el comportamiento del espacio de hojas alrededor de singularidades simples.

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Configuración local de ciclos de campos de vectores analíticos en $\mathbb{R}^3$

N. Corral Pérez [*], M. Martín Vega,[**] F. Sanz Sánchez[**]

En $\mathbb{R}^2$, conocemos todas las posibles configuraciones topológicas locales de las órbitas de un campo de vectores analítico. Si nos centramos en la existencia de ciclos, sabemos que un campo analítico sólo puede definir ciclos en cada entorno arbitrariamente pequeño de la singularidad si tiene configuración de centro, es decir, si existe un continuo de ciclos alrededor de la singularidad del campo. Este resultado se conoce como el "Problema de Dulac", del que se conocen dos demostraciones independientes y altamente no triviales, por parte de Ilyashenko [1] y Écalle [2].

Nos preguntamos cómo será la distribución local de ciclos en $\mathbb{R}^3$ en función de la parte lineal del campo, restringiéndonos a los casos menos degenerados. Si tiene dos o más autovalores con parte real no nula es inmediato probar la ausencia de ciclos. Si tiene un par de autovalores imaginarios, diremos que el campo es una perturbación de un centro lineal no degenerado. En el último caso obtenemos que los ciclos, si existen, solo se pueden organizar localmente en un número finito de superficies topológicas con configuración de centro. El problema sigue abierto para los casos más degenerados.

Agradecimientos A la Universidad de Valladolid por mi contrato predoctoral de la convocatoria 2020 cofinanciada por el Banco Santander y al proyecto de investigación al que estoy vinculada PID2019-105621GB-I00.

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The Seshadri constant for a divisorial valuation

C. Galindo *, F. Monserrat †, C.-J. Moreno-Ávila**, J.-J Moyano-Fernández***

Recently, a valuative Nagata conjecture has been stated in [2, 3]. This conjecture involves a (real) plane valuation ν of the projective plane and a constant named Seshadri-type constant, denoted $\hat{\mu}(\nu)$ and introduced in [1]. Furthermore, the valuative Nagata conjecture implies the Nagata conjecture and asymptotic evidences of its trueness in some particular cases have been proved in [3]. In this talk we introduce a natural Seshadri constant for a smooth projective surface S, a nef divisor on S and a divisorial plane valuation of S and we show some of their properties. These results will help us to give several equivalent geometrical statements to the valuative Nagata conjecture for divisorial plane valuations.

Agradecimientos The authors were partially funded by MCIN/AEI/10.13039/501100011033 and by "ERDF A way of making Europe", grants PGC2018-096446-B-C21 and PGC2018-096446-B-C22, as well as by Universitat Jaume I, grant UJI-B2021-02. The third author was also supported by the Margarita Salas postdoctoral contract MGS/2021/14(UP2021-021) financed by the European Union-NextGenerationEU

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Study of algebraic integrability of foliations on Hirzebruch surfaces. An algorithm of computation depending on the genus.

C. Galindo,* F. Monserrat† E. Pérez-Callejo**

To decide whether a plane foliation has a rational first integral is an open problem. In [1] we present an algorithm that extends a complex planar polynomial vector field $X = A(x,y)(\partial/\partial x) + B(x,y)(\partial/\partial y)$ to a foliation $\mathcal{F}^\delta$ on a Hirzebruch surface with the aim of studying their algebraic integrability.

In this talk, we will show an algorithm that, under certain conditions, decides about algebraic integrability (with generic algebraic invariant curve of fixed genus $g \neq 1$) of $\mathcal{F}^\delta$ which, in the affirmative case, computes a rational first integral. This algorithm uses the reduction of dicritical singularities of a foliation on a Hirzebruch surface.

Agradecimientos Partially supported by MCIN/AEI/10.13039/501100011033 and by “ERDF A way of making Europe", grants PGC2018-096446-B-C22 and RED2018-102583-T, by MCIN/AEI/10.13039/501100011033 and by "ESF Investing in your future”, grant PRE2019-089907, as well as by Universitat Jaume I, grant UJI-B2021-02.

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Computing a difficult invariant of holomorphic map germs

R. Giménez Conejero*

A part of Thom-Mather theory studies deformations of (good) map germs $f : (\mathbb{C}^n, 0) \to (\mathbb{C}^p, 0)$. The case $n < p$ is notoriously more difficult to study, and the case $p = n + 1$ has many long-standing open problems. Images of deformations f_t are of particular interest, as they hide a lot of information about their original germ f. However, they are very difficult to study and are mostly described in topological terms.

We give a way to compute the homology of images of deformations f_t in algebraic terms, specially the image of stable deformations (those who play the role of generic fiber or Milnor fiber for map germs). This allows us to compute the so-called image Milnor number with algebraic invariants, i.e., Milnor numbers of ICIS.

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Interpretación geométrica de la base estándar de una curva plana

D. Senovilla-Sanz [*], F. Cano [**], N. Corral[*]

Dados dos gérmenes de curvas plana en el origen $C_1, C_2 \subset (\mathbb{C}^2, 0)$, decimos que tienen el mismo tipo analítico, si existen dos entornos U_1, U_2 del origen en los que las curvas C_1 y C_2 están definidas respectivamente y un biholomorfismo $\Phi : U_1 \to U_2$ tal que $\Phi(U_1 \cap C_1) = U_2 \cap C_2$.

La clasificación analítica de curvas planas es un problema ya resuelto de forma algorítmica en [2]. Nuestro objetivo es dar información geométrica que complemente este método computacional. Dada una curva irreducible C dada por una parametrización $\phi(t)$, el primer paso para determinar su tipo analítico es calcular su semimodulo de valores diferenciales respecto el semigrupo de la curva C:

$$\Lambda_C = \{\operatorname{ord}_t(\phi^*(\omega)(t)) + 1 : \omega \quad 1 - \text{forma}\}.$$

Este semimódululo Λ_C tiene un sistema minimal de generadores $\lambda_1, \lambda_2, \ldots, \lambda_s$. Una base estandar minimal de la curva C es una familia de 1-formas $\omega_1, \omega_2, \ldots, \omega_s$ tales que $\operatorname{ord}_t(\phi^* \omega_i) + 1 = \lambda_i$.

En este trabajo estudiamos las foliaciones singulares definidas por las 1-formas ω_i cuando la curva C tiene un sólo par de Puiseux. Probamos que dichas foliaciones son dicríticas. Las curvas invariantes de cada una de estas foliaciones correspondientes a la componente dicrítica tienen el mismo tipo topológico que C y las denominamos semirraíces analíticas. Probamos que las semirraíces analíticas codifican parte de la información analítica de la curva C [1].

Presentaremos también algunas de las dificultades encontradas para el caso de curvas con varios pares de Puiseux.

Agradecimientos Los autores están financiados por el proyecto PID2019-105621GB-I00. Este trabajo ha sido parcialmente financiado por la beca predoctoral ”Concepción Arenal"de la Universidad de Cantabria y el Gobierno de Cantabria.

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El Teorema de Comparación Logarítmica y la Euler-homogeneidad en divisores libres

A. Del Valle Rodríguez *

En 2002 se conjeturó que todo divisor libre que satisfaga el Teorema de Comparación Logarítmica ha de ser fuertemente Euler-homogéneo y se dio una prueba en dimensión dos. En 2006 se demostró que la conjetura también es cierta en dimensión tres pero, desde entonces, poco más se ha avanzado en su comprensión y la pregunta sigue abierta en dimensiones superiores. El objetivo de esta charla es explicar en qué consiste esta conjetura y contar mis avances en ella que, hasta el momento, nos han permitido simplificar en gran medida las demostraciones realizadas en dimensión dos y tres.

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Zeta functions, abelian orbifold resolutions of singularities and the geometry of curves in weighted projective planes

J. Viu-Sos*

The Denef-Loeser *topological* and *motivic zeta functions* are analytic invariants of holomorphic map germs $f : (\mathbb{C}^n, 0) \to (\mathbb{C}, 0)$, computed from embedded resolutions of $f = 0$. The Monodromy Conjecture predicts that any pole of the zeta functions is related with an eigenvalue of the monodromy of the Milnor fiber at some point of $f^{-1}(0)$, see e. g. [2, 3].

In this talk, we introduce some recent techniques that we have developed for the study of these zeta functions for $\mathbb{Q}$-divisors over orbifold varieties and compositions of weighted blowing-ups in [1].

Finally, we give some applications of the previous techniques on the computation of poles for some surface singularities via the geometry of curves in embedded weighted projective planes.

This is a joint work Edwin León-Cardenal (Univ. de Zaragoza), Jorge Martín-Morales (Univ. de Zaragoza) and Wim Veys (KU Leuven).

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Geometría Algebraica I: Fundamentos

Organizadores: Alicia Tocino, Alfonso Zamora y Emilio Franco

Lunes 6 de Febrero
Moderadores: *Emilio Franco y Alfonso Zamora*

12:30 *Fibrados principales, grupos no conexos y cubiertas*, Guillermo Barajas.

13:00 *La fibración de Hitchin: análogos y generalizaciones*, Guillermo Gallego.

18:15 *Variedades de caracteres de grupos de nudos tóricos*, Javier Martínez

18:55 *Geometría Sasakiana y superficies complejas*, Juan Rojo

Martes 7 de Febrero
Moderadores: *Emilio Franco, Alicia Tocino y Alfonso Zamora*

10:00 *Embedding superconformal vertex algebras from Killing spinors*, Andoni De Arriba De La Hera

10:30 *Futaki invariants for the Hull-Strominger system*, Raúl González Molina

11:30 *Semimódulos de valoración sobre el anillo local de una rama singular con un par de Puiseux*, Patricio Almirón

12:10 *Continuantes de Euler en geometría de Poisson no conmutativa*, David Fernández

12:50 *Lax colimits of posets with structure sheaves: application to descent*, Javier Sánchez

18:15 *Condition-based Low-Degree Approximation of Real Polynomial Systems*, Josué Tonelli-Cueto

18:55 *Esquemas exponenciables no conmutativos en geometría algebraica plana*, Julia Ramos González

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Semimódulos de valoración sobre el anillo local de una rama singular con un par de Puiseux

Patricio Almirón *

En esta charla caracterizaremos algunos conjuntos de valora- ción de módulos sobre el anillo local de una rama singular con un par de Puiseux. Llamaremos a dichos conjuntos de valora- ción semimódulos crecientes. Veremos que, a partir de la re- presentación planar de los gaps del semigrupo de la curva, po- demos obtener un algoritmo geométrico que permite obtener todos los posibles semimódulos crecientes. En particular, vere- mos que dicho conjunto posee estructura de árbol. Además, la clasificación de estos semimódulos nos permitirá deducir algu- nas propiedades de los módulos de diferenciales de Kahler de la rama. Este es un trabajo conjunto [1] con Maria Alberich- Carramiñana y Julio José Moyano-Fernández.

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Fibrados principales, grupos no conexos y cubiertas

Guillermo Barajas*

Presentamos una equivalencia de categorías entre fibrados principales con grupo de estructura no conexo sobre una variedad topológica X y fibrados principales con grupo de estructura conexo sobre un recubrimiento étale $Y \to X$ equipados con una acción equivariante torcida del grupo de Galois de Y. Una aplicación es la descripción de los puntos fijos de la acción de un grupo finito de la Jacobiana de X en el espacio de moduli de fibrados vectoriales sobre X de rango y grado fijos. En particular, los puntos fijos de tensorizar por un fibrado de línea L de orden finito son pushforwards de fibrados vectoriales sobre un recubrimiento étale determinado por L, lo cual es un resultado clásico de Narasimhan–Ramanan. La principal referencia es mi colaboración con Garcia-Prada–Gothen–Mundet-i-Riera [1].

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Continuantes de Euler en geometría de Poisson no conmutativa

David Fernández*

Originalmente, Euler introdujo en 1764 sus continuantes para estudiar los numeradores y denominadores de las fracciones continuas. Curiosamente, esta familia recursiva de polinomios ha aparecido en diversas e insospechadas ramas de las matemáticas. Por ejemplo, en la teoría de representaciones: Philip Boalch los usó para estudiar ciertas variedades de caracteres salvajes, aparecidas décadas antes en los trabajos de Calabi y Sibuya. De hecho, probó que se pueden describir como variedades de carcajes (quiver varieties) asociadas al carcaj Γ_n de dos vértices y n flechas equiorientadas y poseen una estructura de Poisson (no degenerada).

Como parte de nuestro programa para entender las variedades de caracteres salvajes en términos de la geometría de Poisson no conmutativa de Van den Bergh, en esta ponencia explicaré cómo las estructuras de Poisson encontradas por Boalch están inducidas por ciertas estructuras quasi-Poisson no conmutativas sobre el álgebra de caminos del carcaj Γ_n de manera que los continuantes de Euler corresponden a sus aplicaciones momento (multiplicativas).

Este es un trabajo en colaboración con Maxime Fairon (Orsay).

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La fibración de Hitchin: análogos y generalizaciones

Guillermo Gallego*

La fibración de Hitchin es una aplicación ampliamente estudiada, con dominio el espacio de móduli de G-fibrados de Higgs en una superficie de Riemann, donde G es un grupo reductivo, e imagen un cierto espacio afín; y cuyas fibras son variedades abelianas. Gran parte de las propiedades geométricas de la fibración de Hitchin pueden obtenerse estudiando la aplicación natural del stack cociente del álgebra de Lie de G por su acción adjunta al correspondiente cociente en el sentido de la teoría de invariantes. Esto conduce de forma natural al estudio de aplicaciones de un stack cociente a un cociente GIT en una gran generalidad, que incluye algunos casos de interés para el estudio de varios análogos y generalizaciones de la fibración de Hitchin, como la fibración de Hitchin en dimensión superior, la fibración de Hitchin para pares simétricos, y la fibración de Hitchin multiplicativa.

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Futaki invariants for the Hull-Strominger system

Raúl González*

In a highly influential article [1], Reid put forward a proposal for understanding the moduli space of projective Calabi-Yau threefolds. Roughly, components corresponding to different topologies are glued together via complex surgeries on the manifold, called conifold transitions, so that the resulting moduli is rendered irreducible. Interestingly, this process incorporates non-Kähler backgrounds as 'boundary phenomena' on moduli.

Some years later, Yau proposed that the geometrization of conifold transitions should make use of the Hull-Strominger system, and proposed a conjecture for the existence problem in [4]. This system has its origins in theoretical physics as consistency conditions for heterotic supergravity theories, and is now regarded in hermitian geometry as providing canonical geometry in the non-Kähler realm.

In this talk, we will use methods in generalized geometry as iniciated by Hitchin [3] and Gualtieri [2] to recast the equations of the Hull-Strominger system in terms of a Hermite-Einstein condition on a holomorphic Courant algebroid. Consequently, we will derive a family of subtle holomorphic invariants obstructing the existence of solutions beyond the existence of balanced metrics and stability of bundles. These potentially have the power to decide on Yau's conjecture on the existence problem.

Time permitting, we will discuss a conjectural picture for stability conditions for Courant algebroids which are reminiscent of those in GIT, which makes use of a notion of harmonic metrics. This will be illustrated in a family of examples over homogeneous manifolds.

This is joint work with M. Garcia-Fernandez.

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Embedding superconformal vertex algebras from Killing spinors

Andoni De Arriba De La Hera*

Vertex algebras, introduced by Borcherds to prove the Monstruous Moonshine Conjecture, play an important role in many areas of mathematics, such as representation theory of Kac-Moody algebras and the geometric Langlands correspondence. They have a physical interpretation in two-dimensional conformal field theory, and have had a strong impact in geometry, first by the construction of the chiral de Rham complex by Malikov-Schechtmann-Vaintrob, and recently by the construction of superconformal structures by Heluani-Zabzine among others.

The aim of the talk is to present a method to construct embeddings of the N=2 superconformal vertex algebra, responsible for mirror symmetry, into the chiral De Rham complex. For that, we require solutions of the "Killing spinor equations". These equations come from geometry and physics, specifically from the approach to special holonomy based on generalized geometry on Courant algebroids. To illustrate this, I will give some geometric examples. This talk is based on joint work with Luis Álvarez-Cónsul and Mario Garcia-Fernandez in [1], where we require that the geometry is homogeneous, and in our latest results that will appear in my (very soon) PhD thesis, which has been supervised by them, where there are generalized the previous embeddings for transitive Courant algebroids.

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VI Congreso de Jóvenes Investigadores

Real Sociedad Matemática Española

León, Febrero de 2023

Variedades de caracteres de grupos de nudos tóricos.

Javier Martínez *

Dado un grupo algebraico reductivo complejo G y un grupo finitamente generado Γ, la variedad de caracteres es el cociente GIT

$$X(\Gamma, G) = \mathrm{Hom}(\Gamma, G) /\!\!/ G,$$

que parametriza clases de equivalencia de representaciones de Γ en G, donde G actúa por conjugación. Cuando Γ es el grupo fundamental de una variedad, las variedades de caracteres parametrizan también diversas estructuras geométricas y permiten definir invariantes asociados a ellas, siendo de particular interés analizar el subespacio de representaciones de Γ en K, un compacto maximal de G.

Analizaremos y presentaremos algunos resultados recientes en el caso en que Γ es el grupo de nudos tóricos $\Gamma = \langle a, b \mid a^n = b^m \rangle$ para $\mathrm{SU}(n)$ y $\mathrm{SL}(n, \mathbb{C})$, con atención al caso $n = 3$ (trabajo conjunto con Á.González-Prieto y V.Muñoz).

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VI Congreso de Jóvenes Investigadores
Real Sociedad Matemática Española
León, Febrero de 2023

Esquemas exponenciables no conmutativos en geometría algebraica plana

Ivan Di Liberti* Julia Ramos González**

La geometría algebraica no conmutativa utiliza las categorías de Grothendieck como modelos catégoricos de esquemas potencialmente no conmutativos (véanse, por ejemplo, [9],[8]). Por otro lado, el teorema de Gabriel-Popescu nos permite interpretar las categorías de Grothendieck como los topos enriquecidos en la categoría Ab de grupos abelianos [6]. En concreto, la 2-categoría $\mathsf{Grt}_\flat$ de categorías de Grothendieck y funtores additivos exactos adjuntos por la izquierda, que no es otra que la versión Ab-enriquecida de la 2-categoría de topos y morfismos geométricos, provee a su vez de un ambiente natural para estudiar geometría algebraica plana no conmutativa.

En esta charla mostramos cómo el producto tensorial de Bird-Kelly de categorías Ab-enriquecidas localmente presentables [1][5], que corresponde, por un lado, con el producto de esquemas [2] y por otro lado, con el producto de topos clásico [7], induce una estructura monoidal en $\mathsf{Grt}_\flat$. A continuación, proporcionamos una caracterización de las categorías de Grothendieck exponenciables en $\mathsf{Grt}_\flat$ con respecto a esta estructura monoidal, inspirándonos en la caracterización análoga en la teoría clásica de topos [4]. En particular, este resultado puede verse como una aproximación, desde la geometría algebraica no conmutativa, a la comprensión de la exponenciabilidad de esquemas en el ámbito de la geometría algebraica clásica.

El contenido de esta charla puede encontrarse en [3].

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VI Congreso de Jóvenes Investigadores
Real Sociedad Matemática Española
León, Febrero de 2023

Geometría Sasakiana y superficies complejas.

Juan Rojo Carulli *

En esta charla abordaremos algunas cuestiones dentro del mundo de la geometría algebraica compleja, que vienen motivadas a partir de un problema de geometría diferencial, a saber: ¿existe una variedad de dimensión 5, simplemente conexa, que admita estructuras de K-contacto pero no estructuras Sasakianas?

Dada una variedad Sasakiana de dimensión 5 y simplemente conexa M tal que la torsión de $H_2(M,\mathbb{Z})$ es un p-grupo, se puede construir una superficie compleja X cuya forma de intersección satisface una condición bastante restrictiva: debe existir una $\mathbb{Q}$-base de $H_2(X,\mathbb{Q})$ que se puede representar mediante curvas complejas *disjuntas* de ciertos géneros positivos (preescritos por M). La existencia (o no) de superficies complejas (quizás singulares) que satisfagan esta condición es de importancia fundamental para resolver la primera pregunta que nos planteamos, pero también plantea preguntas que son en sí mismas interesantes dentro de la teoría de superficies algebraicas.

Trataremos de motivar el problema de existencia de una tal superficie compleja y daremos algunos resultados al respecto para el caso en que X es lisa, comentando algunas dificultades extra en el caso de que haya singularidades. Nos basaremos principalmente en resultados obtenidos en [1], [2], [3] y [4].

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VI Congreso de Jóvenes Investigadores
Real Sociedad Matemática Española
León, Febrero de 2023

Lax colimits of posets with structure sheaves: applications to descent.

Javier Sánchez [*]

In this talk we will introduce, for any 1-category or strict 2-category $\mathfrak{C}$, the category of finite posets with a $\mathfrak{C}$-valued «structure» functor: the category of **$\mathfrak{C}$-data**. We will analyze its 2-categorical properties and explicitly compute certain lax colimits of $\mathfrak{C}$-data do not require us to perform any operations on either $\mathfrak{C}$ or the category of posets. These lax colimits will be called «cilinder spaces» for a number of reasons. We will specialize the construction to compute colimits up to some notion of weak equivalence in a subcategory of ringed posets that «models» qc-qs schemes and other more general spaces: the category of schematic finite spaces; and apply the technique it to prove certain descent-like statements, such as a general Seifert-Van Kampen Theorem for an «étale fundamental group» defined on these schematic spaces.

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Real Sociedad Matemática Española

León, Febrero de 2023

Condition-based Low-Degree Approximation of Real Polynomial Systems

J. Tonelli-Cueto,* Elias Tsigaridas†

The most fundamental fact of real algebraic geometry is that the number of real zeros of a generic real polynomial system is not constant. However, what does influence the size of the zero set? In this talk, we show how the condition number of a real polynomial system restrict the possible number of real zeros. As a consequence, we derive important relations between the well-posedness of a real polynomial system and its number of real zeros, and derive new bounds for the number of real zeros of a random real polynomial system.

Agradecimientos Special thanks to Jazz G. Suchen for useful suggestions and moral support.

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Geometría Algebraica II: Aplicaciones y Computación

Organizadores: Josué Tonelli y Angélica Torres

Jueves 9 de Febrero
Moderadores: *Josué Tonelli-Cueto y Angélica Torres*

10:00 *Algebraic reliability of multi-state systems*, Patricia Pascual-Ortigosa

10:30 *Average degree of the essential variety!*, Elima Shehu

11:30 *Context-Specific Decomposable Models*, Eliana Duarte

12:00 *Mu-basis of certain parametrizations in two variables*, Amrutha Balachandran Nair

12:30 *Algebraic Geometry in Optimization*, Lukas Gustafsson

13:00 *In search of energy minimizers on the Grassmannian*, Pedro R. López-Gómez

17:00 *Decomposing minimal problems in computer vision for 3D reconstruction using Galois/monodromy groups*, Margaret Regan

17:30 *Lines in Algebraic Vision*, Felix Rydell

18:00 *Birational Free Form Deformations of degree* $1 \times 1 \times 1$, Pablo Gonzalez Mazon

18:30 *Toric Sylvester forms and applications*, Carles Checa

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León, Febrero de 2023

Mu-basis of certain parametrizations in two variables.

Amrutha Balachandran Nair*

Mu-basis is the basis of syzygy module of a parametrization from which we can retrieve the parametrization back. We will be looking at certain parametrizations in two variables, that describes surfaces and how to find mu-basis. In particular we will be looking at translational surfaces and generalized surfaces of revolution. Translational surfaces, which are surfaces generated by two rational curves by sliding one along the other. Generalized surfaces of revolution are surfaces are generated by rotating a rational space curve around another curve.

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VI Congreso de Jóvenes Investigadores
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León, Febrero de 2023

Toric Sylvester forms and applications

Carles Checa*

The elimination of variables from a system of homogeneous polynomials is deeply connected to the saturation of ideals with respect to a certain geometrically irrelevant ideal. Thus, the search and study of universal generators of the saturation of an ideal generated by generic homogeneous polynomials, such as Jacobian determinants, is an important topic in elimination theory. We investigate this question in the general setting of homogeneous polynomials in the Cox ring of a projective toric variety.

As our main results, we establish a duality property and then we make it explicit by introducing toric Sylvester forms, under a certain positivity assumption on the variety. In particular, we prove that toric Sylvester forms yield a basis of some graded piece of the saturation of I modulo I, where I denotes an ideal generated by $n+1$ generic forms, n is the dimension of the variety. These results generalize classical results of Jouanolou in the case where the variety is a projective space, as well as their recent extension to the case of a product of projective spaces by Buse-Chardin-Nemati. Then, to illustrate the relevance of toric Sylvester forms we provide some consequences in elimination theory.

First, we define hybrid elimination matrices by incorporating toric Sylvester forms in the classical Macaulay matrices; more generally, from the classical Koszul complex we obtain a new complex whose determinant of some of its graded pieces is proved to be equal to the toric resultant in the fashion of Cattani-Dickenstein-Sturmfels for the Jacobian case.

Second, we extend the construction of hybrid elimination matrices to the case of overdetermined polynomial systems, providing this way a new family of matrices that can be used for solving such polynomial systems by means of linear algebra methods. Finally, we show that one can take advantage of toric Sylvester forms for computing toric residues of product of forms; this is a generalization of a formula due to Jouanolou for the projective space.

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León, Febrero de 2023

Context-Specific Decomposable Models

Eliana Duarte*

In this talk I will introduce a class of context-specific independence statistical models and use algebra to characterize all the context-specific independence relations that the model satisfies. These models are a generalization to the context-specific setting of the widely popular decomposable graphical models and we prove they exhibit similar properties.

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León, Febrero de 2023

Birational Free Form Deformations of degree $1 \times 1 \times 1$

Pablo Gonzalez Mazon*

Free Form Deformation (FFD) stands for a simple and intuitive method for the geometric manipulation of 2D and 3D objects of arbitrary shape, which has multiple applications in Computer Aided Geometric Design (CAGD) and related fields. It consists of the transformation of the ambient space following a rational map that roughly captures the desired geometry. To be considered valid, this rational map must be injective on the object. More interestingly, birational maps are globally injective and allow the exact computation of preimages avoiding numerical methods, which is convenient for most applications.

Effective tools for the manipulation of 2D FFDs given by a birational map have already been developed, and provide provide a similar experience to a user in comparison to classical 2D FFDs. Concerning 3D FFDs, there are still no efficient criteria that are flexible enough to be used for modeling purposes.

In this talk, we provide effective methods for the manipulation of birational 3D FFDs of degree $1 \times 1 \times 1$. Geometrically, these transformations are determined by two control points in each direction of the space (i.e. $2 \times 2 \times 2 = 8$ control points), and their associated weights. We explain how the geometry of the control points needs to be constrained, and provide explicit formulas for the weights and the inverse rational maps.

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León, Febrero de 2023

Algebraic Geometry in Optimization

Lukas Gustafsson*

A common problem in applications is fitting data to a model. This requires an objective function that acts as a notion of 'distance' to points in and outside your model. Our main examples of non-linear objective functions are the Euclidean distance and the log-likelihood functions, which we then restrict to semi-algebraic sets. When passing over to the complex numbers, the number of critical points of these functions is pre-determined, regardless of the input data. This brings us to the topic of the talk: what are these numbers and how can we compute them? Can we classify the statistical models where there is a unique complex critical point?

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León, Febrero de 2023

In search of energy minimizers on the Grassmannian

Pedro R. López-Gómez*

In recent years, the problem of generating well-distributed collections of points on compact Riemannian manifolds has attracted the interest of both the pure and applied mathematical community. A frequently used approach to this problem consists on considering the collections that minimize a certain pairwise interaction energy. In this talk, I will focus on the Grassmannian $\mathrm{Gr}_{m,n}(\mathbb{K})$, that is, the set of m-dimensional linear subspaces of $\mathbb{K}^n$, where $\mathbb{K} = \mathbb{R}$ or $\mathbb{C}$. This space has received considerable attention in fields like information theory or computer vision, but little is known about the energy minimizers. I will introduce this problem, with particular emphasis on the case $\mathrm{Gr}_{2,4}(\mathbb{R})$.

This is a joint work with Ujué Etayo.

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León, Febrero de 2023

Algebraic reliability of multi-state systems

Patricia Pascual-Ortigosa*

In this talk we will explore two different methods based on monomial ideals for computing the reliability of multi-state systems. The first one is based on multigraded Hilbert series and, the second one, in involutive basis. We will show computational examples contrasting both approaches.

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VI Congreso de Jóvenes Investigadores
Real Sociedad Matemática Española
León, Febrero de 2023

Decomposing minimal problems in computer vision for 3D reconstruction using Galois/monodromy groups

Margaret Regan*

Many problems in computer vision are represented using a parameterized polynomial system, where the solution set is critical for 3D reconstruction. Minimal problems can be of special interest as these polynomial systems are well-constrained and generically have finitely many solutions. Computing the Galois/monodromy group of the associated branch cover can yield information and understanding about the underlying structure of the minimal problem. One potential outcome of this computation is identifying possible decompositions or symmetries that enable more efficient solution solving. These Galois/monodromy groups can be computed using numerical homotopy continuation via a multitude of softwares. Examples such as the classical homography estimation and five-point relative pose problems will be provided to show these computations and the structure they can help identify. This is joint work with Timothy Duff, Viktor Korotynskiy, and Tomas Pajdla.

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León, Febrero de 2023

Lines in Algebraic Vision

Felix Rydell*

Algebraic vision is the application of algebraic geometry to reconstruction problems in computer vision. Given a fixed set of cameras that have taken pictures of an object, the goal is to reconstruct a 3D model of the object. Triangulation is the act of using point and line correspondences to reconstruct a point and line cloud for this model. In this talk we discuss line multiview varieties, triangulation using lines and related Euclidean distance degrees.

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Average degree of the essential variety!

Elima Shehu*

The essential variety is an algebraic subvariety of dimension 5 in $\mathbb{R}P^3$, which encodes the relative pose of two calibrated cameras. A calibrated camera is a matrix $C = [R, t]$, where $R \in SO(3)$ and $t \in \mathbb{R}^3$. Since the degree of this variety is 10, there can only be at most 10 complex solutions. We compute the expected number of real points in the intersection of the essential variety with a random linear space of codimension 5. In this talk, I will tell you about these computations and our results. This is a joint work with Paul Breiding, Samantha Fairchild and Pierpaola Santersiero.

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Teoría de Grafos Extremales y Aleatorios

Organizadores: Alberto Espuny Díaz

Martes 7 de Febrero
Moderador: *Alberto Espuny Díaz*

11:30 *Resilience for loose Hamilton cycles*, Richard Lang

12:10 *Maximum running times for graph bootstrap percolation processes*, Patrick Morris

12:50 *Transversal cycle factors in multipartite graphs*, Amarja Kathapurkar

Miércoles 8 de Febrero
Moderador: *Alberto Espuny Díaz*

11:30 *How to build a pillar: a proof of Thomassen's conjecture*, Irene Gil Fernández

12:10 *Hypergraphs with minimum uniform Turán density*, Ander Lamaison

12:50 *Ramsey numbers with prescribed rate of growth*, Simón Piga

Jueves 9 de Febrero
Moderador: *Alberto Espuny Díaz*

17:00 *Preservation theorems on random graphs*, Alberto Larrauri

17:40 *The expected number of perfect matchings in cubic planar graphs*, Clément Requilé

VI Congreso de Jóvenes Investigadores

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León, Febrero de 2023

How to build a pillar: a proof of Thomassen's conjecture

A. Irene Gil Fernández* B. Hong Liu †

Carsten Thomassen in 1989 conjectured that if a graph has minimum degree much more than the number of atoms in the universe ($\delta(G) \geq 10^{10^{10}}$), then it contains a *pillar*, which is a graph that consists of two vertex-disjoint cycles of the same length, s say, along with s vertex-disjoint paths of the same length[1] which connect matching vertices in order around the cycles. Despite the simplicity of the structure of pillars and various developments of powerful embedding methods for paths and cycles in the past three decades, this innocent looking conjecture has seen no progress to date. We have recently given a proof of this conjecture by building a pillar (algorithmically) in sublinear expanders. The reason why we use such graphs is that every graph contains a robust expander subgraph with almost the same average degree, so we can assume that our initial graph is a robust expander and enjoy its expansion properties to find a pillar.

Our proof uses a key structure called *kraken*. Roughly speaking, a kraken consists of a cycle, in which every vertex has a large 'boundary'. If we manage to find two krakens, then we can link the matching vertices in their cycles by expanding and connecting their boundaries to obtain a pillar. Finding a *single* kraken in a sublinear expander is already not an easy task; this was done in [1] with an involved argument. To find two krakens here, we prove a *robust* embedding lemma for kraken, which is the main challenge and contribution of [2]. We expect it to have further applications for embedding problems. Its proof uses the existence of kraken in sublinear expanders from [1] as a black box and builds on the techniques developed in the work of Liu and Montgomery [3].

More specifically, we split the proof into two steps. The first one consists of finding a kraken robustly, i.e., we prove that there exists a kraken with the required properties while avoiding a connected set of vertices (which, in the proof of the main result, is the first kraken that we do not want to touch, so the two krakens are disjoint). In the second step we show that we can link each pair of vertices in the cycles inside krakens via their legs disjointly to construct a pillar. The key point here is that we need to make sure that the paths that we build are all of the same length. In order to do this, we use an object introduced by Liu and Montgomery in [3] called *adjusters*. An adjuster consists of an even cycle C, together with two disjoint large connected subgraphs F_1, F_2 attached to two almost-antipodal vertices v_1, v_2 on the cycle C. If C has length 2ℓ, for some $\ell \leq \log n$, there are two v_1, v_2-paths, one of length $\ell + 1$ and the other with length $\ell - 1$. The subgraphs F_1, F_2 are set to be comfortably larger than the size of C, so that they can be connected by a short path while avoiding C. The idea is to link many such structures sequentially to form an *adjuster*, so that we can use it to find paths of many different lengths by varying the length of the path we take around each cycle.

Acknowledgements We would like to thank Richard Montgomery for fruitful discussions

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H.L. was supported by the Institute for Basic Science (IBS-R029-C4) and the UK Research and Innovation Future Leaders Fellowship MR/S016325/1.

[1] Note that the length of these paths is not necessarily s (the length of the cycles).

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León, Febrero de 2023

Transversal cycle factors in multipartite graphs

Amarja Kathapurkar[*] Richard Mycroft [†]

In this talk, we discuss a generalisation of tilings into the multipartite setting. In particular, let G be a k-partite graph with parts $V_1, \dots, V_k$ each of size n, and for each $i \in [k]$, let the minimum degree in the bipartite graph $G[V_i, V_{i+1}]$ be at least $(1+1/k)n/2$. We show that if k is even, G contains a C_k-factor in which each copy of C_k contains exactly one vertex from each V_i, that is, a *transversal* C_k-factor. This resolves independent conjectures of Fischer and Häggkvist in the case when k is even. When k is odd, we show that either G contains a transversal C_k-factor or G is 'close to' being extremal.

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Hypergraphs with minimum uniform Turán density

Ander Lamaison*

Reiher, Rödl and Schacht showed that the uniform Turán density of every 3-uniform hypergraph is either 0 or at least $1/27$, and asked whether there exist 3-uniform hypergraphs with uniform Turán density equal or arbitrarily close to $1/27$. We construct 3-uniform hypergraphs with uniform Turán density equal to $1/27$.

Joint work with Frederik Garbe and Dan Král.

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León, Febrero de 2023

Resilience for loose Hamilton cycles

J. Alvarado[*], Y. Kohayakawa[*], R. Lang[†], G. Mota[*], E. Stagni[*]

We study the emergence of loose Hamilton cycles in subgraphs of random hypergraphs. Our main result states that the minimum d-degree threshold for loose Hamiltonicity relative to the random k-uniform hypergraph $\mathcal{H}_k(n,p)$ coincides with its dense analogue whenever $p \geq n^{-(k-1)/2+o(1)}$. The value of p is approximately tight for $d > (k+1)/2$. This is particularly interesting because the dense threshold itself is not known beyond the cases when $k = 3$ or $d \geq k-1$.

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VI Congreso de Jóvenes Investigadores

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León, Febrero de 2023

Preservation theorems on random graphs

Alberto Larrauri *

Preservation theorems are classical results about first-order (FO) logic, that show equivalence between several semantic and syntactic classes of formulas. For instance, Lyndon's theorem states that any increasing property of graphs (both finite and infinite) that can be expressed with FO logic, can be expressed by a positive sentence. Similarly, the Łoś-Tarski Theorem says that any FO-expressible property of graphs (both finite and infinite) that is preserved by taking induced subgraphs can be expressed by some universal sentence. Another result is the Homomorphism Preservation Theorem.

Similarly to other central theorems about FO logic, such as the Compactness Theorem, both Lyndon's and Łoś-Tarski theorems fail when restricted to finite structures. For example, there exist FO-expressible properties that are increasing over finite graphs, but cannot be written as a positive sentence.

The counterexamples for the finite restrictions of Lyndon's and Łoś-Tarski theorems are fairly involved and, arguably, artificial. In this regard, there is a research line that studies which additional restrictions are sufficient to recover preservation theorems. For instance, it is known that the Łoś-Tarski Theorem holds in any class of finite graphs with bounded treewidth.

We study preservation theorems on finite structures from a probabilistic standpoint. Given a sequence of random graphs $\mathcal{G}_n$, we say that Lyndon's Theorem holds with high probability if for any FO-expressible property P which is increasing over finite graphs there is a positive sentence φ such that the set of graphs where P and φ are not equivalent has limit probability zero. We also define a probabilistic version of the Łoś-Tarski Theorem.

In this work, we ask whether the probabilistic versions of Lyndon's and Łoś-Tarski theorems hold in some known models of random graphs and we obtain several positive results. We examine the classical Erdős-Rényi graph $\mathcal{G}(n,p)$ in three different ranges of $p = p(n)$: (1) the very sparse range, where $p(n) \sim cn^{(\ell+1)/\ell}$ with ℓ some positive integer, (2) the sparse range, where $p(n) \sim c/n$, and (3) the connectivity threshold, where $p(n) = \Theta(\ln n/n)$. We also study uniform random graphs from addable minor-closed classes.

Our main results are that the probabilistic versions of Lyndon's and Łoś-Tarski theorems hold in all the ranges of $\mathcal{G}(n,p)$ we consider, and the probabilistic Łoś-Tarski Theorem is satisfied by uniform graphs in addable minor-closed classes.

Acknowledgments I wish to thank Albert Atserias and my supervisor Marc Noy for our helpful discussions on the logic of random graphs.

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VI Congreso de Jóvenes Investigadores

Real Sociedad Matemática Española

León, Febrero de 2023

Maximum running times for graph bootstrap percolation processes

David Fabian[*] Patrick Morris[†] Tibor Szabó[*]

Given a fixed graph H and an n-vertex graph G, the H*-bootstrap percolation process* of H on G is defined to be the sequence of graphs G_i, $i \geq 0$, which starts with $G_0 := G$ and in which G_{i+1} is obtained from G_i by adding every edge that completes a copy of H. This process is an example of a cellular automata and has been extensively studied since being introduced by Bollobás [2] in 1968. Recently, Bollobás raised the question of determining the maximum running time of this process, over all choices of n-vertex graph G. Here, the running time of the process is number of steps t the process takes before stabilising, that is, when $G_t = G_{t+1}$. Recent papers of Bollobás–Przykucki–Riordan–Sahasrabudhe [3], Matzke [4] and Balogh–Kronenberg–Pokrovskiy–Szabó [1] have addressed the case when H is a clique, and determined the asymptotics of this maximum running time for all cliques apart from K_5. Here, we initiate the study of the maximum running time for other graphs H and provide a survey of our new results in this direction. We study several key examples, giving precise results for trees and cycles, and giving general results towards understanding how the maximum running time of the H-bootstrap percolation process depends on properties of H, in particular exploring the relationship between this graph parameter and the degree sequence of H. Many interesting questions remain and, along the way, we indicate some directions for future research.

Agradecimientos DF supported by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) Graduiertenkolleg "Facets of Complexity" (GRK 2434). PM supported by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) Walter Benjamin program - project number 504502205. TSz supported by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) under Germany's Excellence Strategy - The Berlin Mathematics Research Center MATH+ (EXC-2046/1, project ID: 390685689).

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VI Congreso de Jóvenes Investigadores

Real Sociedad Matemática Española

León, Febrero de 2023

Ramsey numbers with prescribed rate of growth

Matías Pavez-Signé * Simón Piga† Nicolás Sanhueza-Matamala‡

Let $R(G)$ be the two-colour Ramsey number of a graph G. In this note, we prove that for any non-decreasing function $n \leq f(n) \leq R(K_n)$, there exists a sequence of connected graphs $(G_n)_{n\in\mathbb{N}}$, with $|V(G_n)| = n$ for all $n \geq 1$, such that $R(G_n) = \Theta(f(n))$. Unexpectedly, we also show that an analogous statement does not hold for hypergraphs of uniformity at least 5.

We use our techniques to answer a question posed by DeBiasio about the existence of sequences of graphs whose 2-colour Ramsey number is linear whereas their 3-colour Ramsey number has superlinear growth.

We discuss further questions and open problems.

Agradecimientos MPS was supported by the European Research Council grant 947978 under the European Union's Horizon 2020 research and innovation programme. SP was supported by EPSRC grant EP/V002279/1. NSM was partly supported by ANID-Chile through the FONDECYT Iniciación N°11220269 grant.

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VI Congreso de Jóvenes Investigadores

Real Sociedad Matemática Española

León, Febrero de 2023

The expected number of perfect matchings in cubic planar graphs

Marc Noy [*] Clément Requilé [†] Juanjo Rué [‡]

In the 1970s, Lovász and Plummer [3] famously conjectured that a bridgeless cubic graph on n vertices has exponentially many perfect matchings. In 2011, it was finally proven by Esperet et al. [2] that the number of perfect matchings is at least $2^{n/3656} \simeq 1.0002^n$. While in 2012, Chudnovsky and Seymour [1] showed that if you restrict yourself to the special case of bridgeless cubic planar graphs, you get at least $2^{n/655978752} \simeq 1.000000001^n$ perfect matchings.

In this talk, we consider uniform random bridgeless cubic planar graphs with n labeled vertices. Under this model, we show [4] that the expected number of perfect matchings is asymptotically $c\gamma^n$, where $c > 0$ and $\gamma \simeq 1.14196$ is an explicit algebraic number. We also compute it for (not necessarily bridgeless) cubic planar graphs and provide lower bounds for unlabeled graphs. Our starting point is a correspondence between perfect matchings in rooted cubic planar maps, that are embeddings of planar graphs, and the partition function of the Ising model in rooted triangulations of the sphere.

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Investigación Operativa

Organizadora: Marina Leal

Lunes 6 de Febrero
Moderadores: *Consuelo Parreño y Víctor España*

12:30 *Sustainable last-mile delivery using public transport*, Alberto Santini.

13:00 *A mathematical model for the Home Health Care Routing and Scheduling Problem*, Consuelo Parreño Torres.

18:15 *The Hampered Traveling Salesman Problem with Neighbourhoods*, Carlos Valverde

18:45 *A Branch and Price for the Min-Max Close-Enough Arc Routing Problem*, Miguel Reula

19:15 *El problema del cartero rural con una flota de drones*, Paula Segura

Martes 7 de Febrero
Moderadores: *Laura Davila Pena, Alberto Torrejón Valenzuela y Francisco Temprano*

10:00 *Features of the λ-cent-dian solution network*, Natividad González

10:30 *A new source connection problem*, Laura Davila Pena

11:30 *The α-CVaR and other alternatives to deal with Uncertainty in the Hybridized Maximal Covering Location Problem*, Ricardo Gázquez

12:00 *On Collaboration in Multi-period Discrete Location*, Carmen Ana Domínguez Bravo

12:30 *Optimizing Treatment Allocation to Maximize the Health of a Population*, Alba Olivares

13:00 *Improving algorithm efficiency for fairness allocation in optimization*, Alberto Torrejón Valenzuela

18:15 *Optimización de la asignación de atraques considerando el tiempo de movimiento de las grúas*, Juan Francisco Correcher

18:40 *Estimating Ensemble weights for Bagging Regressors based on the Mean-Variance portfolio framework*, Thomas Ashley

19:00 *Single-linkage clustering with feature selection: a mathematical optimization approach*, Marina Leal

19:20 *A Mathematical Programming approach to the Minimum Normalized Cut Problem*, Francisco Temprano

Miércoles 8 de Febrero
Moderadora: *Paula Segura*

10:00 *The Boosted DC Algorithm for linearly constrained DC programming*, Rubén Campoy

10:30 *Un modelo aditivo basado en funciones lineales a trozos y aprendizaje automático para la estimación de funciones de producción*, Víctor España

VI Congreso de Jóvenes Investigadores

Real Sociedad Matemática Española

León, Febrero de 2023

Estimating Ensemble weights for Bagging Regressors based on the Mean-Variance portfolio framework

Javier Pérez-Rodríguez*, Francisco Fernández-Navarro*, Thomas Ashley*

In this work, we consider a novel ensemble learning framework inspired by modern portfolio optimization in order to address regression problems. Ensemble learning in a mean-variance framework allows for the determination of ensemble's weights considering the error predictions of the base learners, the variability in their predictions, and the covariance among their predictions' errors, which is aligned with the bias-variance-covariance theory ([1]). Under the framework proposed, four potential instantiations have also been provided. The first two ensemble models impose the non-negativity constraints on the ensemble's weights (along with the equality constraint that the ensemble's weights sum up to one) and are solved with the active set algorithm. The second two ensemble models do not include the non-negativity constraints on the ensemble's weights (which in the financial literature are called the non-shorting constraints), giving rise to a convex quadratic programming (QP) problem (as the matrix included in the quadratic term is symmetric and positive definite) that is solved by the Lagrangian procedure. Extensive experiments with regression datasets evaluate the proposed ensemble framework. Comparisons with other state-of-the-art ensemble methods ([2],[3]) confirm that the ensemble framework yields the best overall performance.

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The Boosted DC Algorithm for linearly constrained DC programming

Rubén Campoy* Francisco J. Aragón Artacho† Phan T. Vuong‡

The Boosted Difference of Convex functions Algorithm (BDCA) has been recently introduced in [1, 2] to accelerate the performance of the classical Difference of Convex functions Algorithm (DCA). This acceleration is achieved thanks to an extrapolation step from the point computed by DCA via a line search procedure. However, this line search will only terminate finitely when the first function in the DC decomposition is differentiable. Thus, BDCA cannot be efficiently applied to constrained DC problems. In this talk, we present an extension of BDCA [3] that can be applied to DC programs with linear constraint and we show its convergence under mild assumptions. We also present some numerical experiments where we compare the performance of DCA and BDCA on some challenging problems.

Agradecimientos This work was partially supported by the Ministry of Science, Innovation and Universities of Spain and the European Regional Development Fund (ERDF) of the European Commission (PGC2018-097960-B-C22), and by the Generalitat Valenciana (AICO/2021/165).

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VI Congreso de Jóvenes Investigadores

Real Sociedad Matemática Española

León, Febrero de 2023

Optimización de la asignación de atraques considerando el tiempo de movimiento de las grúas

J.F. Correcher * F. Perea † R. Alvarez-Valdes *

La logística portuaria es hoy en día un campo de estudio de gran importancia en el ámbito de la Investigación Operativa. En las terminales de contenedores, el operador trata de optimizar diversas operaciones para reducir los costes de la terminal y ofrecer un servicio de carga y descarga rápido a las navieras. En la zona del muelle se debe optimizar el uso de la línea de atraque, cuya longitud es limitada, y las grúas existentes, que comparten un mismo raíl y, por tanto, no pueden entrecruzarse. El problema de planificación del uso de atraques y grúas por los barcos ha sido ampliamente estudiado [1, 2], pero los enfoques seguidos hasta la fecha han ignorado el impacto del tiempo de movimiento de las grúas entre barcos y su tiempo de preparación para iniciar las operaciones. En esta ponencia presentamos un modelo lineal entero y un algoritmo genético que hemos desarrollado específicamente para resolver este problema. Los experimentos realizados muestran que el modelo, ejecutado en un solver, permite resolver óptimamente problemas de hasta 40 barcos esperados en una semana y que las soluciones obtenidas son más realistas y eficientes. Por su parte, el algoritmo genético permite obtener soluciones factibles de conveniencia bajo restricciones de tiempo de computación para problemas de cualquier tamaño de interés práctico.

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VI Congreso de Jóvenes Investigadores
Real Sociedad Matemática Española
León, Febrero de 2023

A new source connection problem

L. Davila-Pena*,† P. Borm‡ I. García-Jurado*§ J. Schouten¶

This work focuses on a new cooperative operational research problem that arises by combining ideas from the minimum cost spanning tree problems and sequencing problems: the graph machine scheduling problem (GMS-problem, [1]). It can be motivated by a situation of water distribution from a central reservoir to different locations in an urban area, which correspond to agents. The objective pursued comprises two stages: first, to design a network for supplying all the agents at minimum cost, and second, to allocate the cost of the optimal network among all the agents involved.

We will present a detailed description of how to obtain an optimal solution to address the first of the goals for certain networks, and we will give some hints on how to tackle the second of the above questions.

Acknowledgments Laura Davila-Pena's research has been supported by the former Spanish Ministerio de Educación, Cultura y Deporte through the contract FPU17/02126 and by the Spanish Ministerio de Universidades through mobility grant EST21/00360. This work is part of the R+D+I project grant PID2021-124030NB-C32, funded by MCIN/AEI/10.13039/501100011033/ and by "ERDF A way of making Europe"/EU. This research was also funded by Grupo de Referencia Competitiva ED431C 2021/24 from the Consellería de Cultura, Educación e Universidades, Xunta de Galicia.

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VI Congreso de Jóvenes Investigadores

Real Sociedad Matemática Española

León, Febrero de 2023

On Collaboration in Multi-period Discrete Location

C. Dominguez-Bravo[*]   L. F. Escudero[**]   E. Fernandez[*]   J. F. Monge[***]

In this work, we study discrete location models in which several agents operate on a common network and time horizon. We consider cost-minimization models, where the existing demand at each time-period must be served, as well as prize-collecting extensions including decisions on the users to serve. All models allow multiple-allocation of users to open facilities, which may vary throughout the time horizon. We operate on a modular capacity context in which the initial capacity and potential future extensions are part of the decision-making process.

We compare a non-collaborative setting in which each agent makes its own location, facility capacity, and service decisions, with flexible collaborative policies in which agents make joint decisions. According to such policies, location and service decisions are no longer linked to their original agents. That is, at any time-period a plant originally “belonging” to one agent can be opened and operated by a different one. Furthermore, the demand of a user originally relative to a given agent may be served by a different one, which may change among time-periods. Alternative mathematical programming formulations are proposed both for the non-collaborative setting and for each of the considered collaborative policies.

Benchmark instances from the literature have been adapted to the new models and used in a series of computational experiments, whose preliminary numerical results are summarized. The effectiveness of the alternative formulations is analyzed. Finally, we compare the performance of the collaborative models versus the non-collaborative ones, and show the empirical advantages that agent collaboration may produce.

Agradecimientos This research is partially supported by the Spanish Ministry of Science and Innovation through projects MTM2019-105824GB-I00 and RED2018-102363- T, and by the Junta de Andalucía Regional through project FQM270.

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VI Congreso de Jóvenes Investigadores
Real Sociedad Matemática Española
León, Febrero de 2023

Un modelo aditivo basado en funciones lineales a trozos y aprendizaje automático para la estimación de funciones de producción

V. España,* J. Aparicio* X. Barber* M. Esteve*

El campo del análisis de eficiencia y productividad se centra en la estimación de fronteras de producción y medición de eficiencia de un conjunto de unidades homogéneas (empresas o cualquier otro tipo de organización) llamadas, de forma genérica, *Decision Making Unit* (DMU) que utilizan cierta cantidad de *input* para la producción de uno o varios *outputs*. Las metodologías existentes para este propósito pueden ser clasificadas como paramétricas o no paramétricas. Entre las técnicas no paramétricas, *Data Envelopment Analysis* (DEA) ([1], [2]) es una de las más extendidas en la literatura.

En este contexto, el artículo introduce una nueva metodología no paramétrica cuyo fin es la estimación de funciones de producción (consideran la producción de un único *output*) que satisfacen ciertos axiomas clásicos de la teoría de la producción en microeconomía como la monotonicidad y la concavidad, mediante la adaptación de la versión aditiva de la técnica de aprendizaje automático Multivariate Adaptive Regression Splines (MARS) introducida en [3]. Este nuevo enfoque comparte con DEA la forma del predictor como una función lineal a trozos, aunque nuestra metodología también puede presentarse como un predictor suavizado (con primeras derivadas continuas). Por otro lado, nuestro nuevo enfoque supera los problemas de sobreajuste presentes en DEA haciendo uso de la técnica de validación cruzada generalizada y mediante la selección óptima de un conjunto de hiperparámetros que permiten controlar la complejidad de los modelos construidos. De este modo, se consigue un modelo muy flexible que se adapta a escenarios con estructuras muy diversas.

La nueva metodología ha sido evaluada mediante una serie de experimentos computacionales para medir su desempeño frente a otras técnicas del campo del análisis de eficiencia como el propio DEA u otras metodologías más recientes en el tiempo, como *Corrected Concave Non-Parametric Least Squares* (CCNLS) ([4], [5]). Los resultados obtenidos muestran que nuestra técnica es capaz de proporcionar errores medios y sesgos menores que estas metodologías, aunque se ha podido comprobar que su éxito dependerá en gran medida del nivel de interacción de las variables y del grado de no aditividad de la verdadera función de producción estimada.

Agradecimientos Los autores agradecen la subvención PID2019-105952GB-100 financiada por el Ministerio de Ciencia e Innovación/Agencia Estatal de Investigación/10.13039/501100011033. V. España agradece el apoyo económico de la Generalitat Valenciana en el marco de la beca ACIF/2021/135.

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VI Congreso de Jóvenes Investigadores

Real Sociedad Matemática Española

León, Febrero de 2023

The α-CVaR and other alternatives to deal with Uncertainty in the Hybridized Maximal Covering Location Problem

Víctor Blanco,[*] Ricardo Gázquez[†] Francisco Saldanha-da-Gama[‡]

Our work lies in the area of maximal covering facility location which is a well-defined branch of Location Science and it firstly proposed in [2]. In [1], the authors motivate the use of different types of facilities in maximal covering location problems by applications in telecommunications or surveillance activities.

In this talk, we specialize the above work by considering a two-stage maximal covering location problem under uncertainty. In the first stage, a decision has to be made about a set of initial facilities to locate. These can be selected in a finite set—a discrete facility location setting is adopted. In the second stage, a set of additional locations are selected in the whole space underlying the problem assumed to be $\mathbb{R}^d$, for some $d \geq 2$. Additionally we assume that the exact number of second-stage facilities to locate is not known beforehand—it corresponds to information to be disclosed in future moment in time, for instance, depending on a budget whose volume is still unknown. The overall problem is cast as a maximal covering location problem with two types of facilities and uncertainty associated with the maximum number of second-stage facilities that can be opened.

There are multiple possibilities to capture risk or uncertainty in Facility Location. In case one is not provided with probability distributions on the random parameter (the number of extra facilities to be located in the future) one can assort to many measures of robustness proposed for this situation, being the two most common for a maximization problem the *maximin* and the *maximin regret*. On the other hand, in case one is provided a probability distribution for the random parameter, the most popular approach is maximizing the mean outcome of the system, in this case maximizing the *expected* coverage of the facilities (the so called expected coverage model).

A different probabilistic approach is the notion of α-reliability sets as a subset of scenarios with joint probability at least a given threshold $\alpha \in [0, 1]$, and that can be incorporated to the previously described models, which, will make decisions take into account only the scenarios *selected* in the α-reliability sets. With this idea, the α conditional value at risk (α-CVaR) approach minimizes a convex combination of the maximum regret with respect to the scenarios in an α-reliability set and the expected regret with respect to the remainder scenarios.

In this talk we will focus on the α-CVaR model, that, as far as we know has been never incorporated to MCLP-based problems, and whose model require a deeper effort to derive a mathematical optimization based approach on the underlying model.

Agradecimientos This talk has been partially supported by Spanish Ministerio de Ciencia e Innovación, AEI/FEDER grant number PID2020-114594GBC21 and B-FQM-322-UGR20.

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VI Congreso de Jóvenes Investigadores

Real Sociedad Matemática Española

León, Febrero de 2023

Features of the λ-cent-dian solution networks

Víctor Bucarey* Natividad González-Blanco† Martine Labbé‡ Juan A. Mesa†

The term λ-*cent-dian* was first introduced for facility location problems with $\lambda \in [0,1]$ in [1], which consists of a linear convex combination of center and median objectives. Then, authors in [2] introduced the concept of *generalized-center* as the minimizer of the difference function between the center and the median. This turns out to be the particular case in which we have $\lambda \to \infty$.

The λ-cent-dian objective has also been considered in extensive facility location problems (see [4], [6]). The first time that a research on λ-cent-dian extensive facilities was published was in [3].

Subsequently, we have extended the existing notions of λ-cent-dian and generalized-center to the area of Network Design. For that, we consider the problems of building a subnetwork of an underlying network under a budget constraint to serve a set of origin/destination pairs instead of single points. These problems are focused on designing a network that minimizes the maximum distance of a known set of origin-destination pairs (within that network), the average distance, a linear combination of both objectives or the difference between them. These objectives may be of interest for some of the needs at the present time.

We prove and show that situations similar to those presented in [5] occur when trying to solve the problem of designing a network instead of locating a facility. The λ-cent-dian problem study the trade-off between efficiency and equity. We investigate the properties of the λ-cent-dian and generalized-center solution networks under the lens of equity, efficiency, and Pareto-optimality. Furthermore, we evaluate the quality of the different solution concepts using some inequality measures.

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VI Congreso de Jóvenes Investigadores

Real Sociedad Matemática Española

León, Febrero de 2023

Single-linkage clustering with feature selection: a mathematical optimization approach

Martine Labbé* Mercedes Landete† **Marina Leal**‡

Single-linkage clustering is a type of agglomerative hierarchical clustering method whose output is a tree called Dendrogram. These dendrograms are representations of the two-dimensional cluster similarity matrix. A result in the literature shows that dendrograms generated by the single-linkage method can be obtained by solving the Minimum Spanning Tree problem [2]; all the information required for this type of dendrogram of a set of points is contained in a Minimum Spanning Tree. These representations are useful in a wide variety of fields such as medicine, biology, anthropology or sociology among others. Feature selection is common in data mining for managing large databases (see, for instance, [1, 3] or [4], and the references therein). Since dendrograms represent groupings of large databases, they are also likely to the use of feature selection. In this work, we introduce a bilevel model for feature selection in minimum spanning trees, for different objective functions and analyze their polyhedral properties. We also introduce theorems that allows the statement of a decomposition algorithm for solving some of the proposed problems. We show the benefit of the properties and the algorithm with a computational study.

Agradecimientos This work was partially supported by the Spanish Ministry of Science and Innovation through projects PGC2018-099428-B-100, RED2018-102363-T and PID2020-114594GB-C21, PID2021-122344NB-I00, also by CIGE/2021/161 and PROMETEO/2021/063 funded by the governments of Spain and the Valencian Autonomous Region, and by P18-FR-1422 and US-1256951 funded by Junta de Andalucía and FEDER.

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VI Congreso de Jóvenes Investigadores
Real Sociedad Matemática Española
León, Febrero de 2023

Optimizing Treatment Allocation to Maximize the Health of a Population.

Alba V. Olivares Nadal[*] Daniel Adelman[†]

We use Markov Decision Processes (MDPs) and Dynamic Programming to model the task of optimally allocating treatment amongst patients while fulfilling some capacity constraints. The complexity of such a problem may be very high because health-care populations may be large enough that gathering information of the current status of each patient and tracking the evolution of their covariates is untenable. To address this challenge we develop the so-called measurized theory, which allows to model MDPs that optimize the distribution of treated and untreated patients instead of dealing with identifed patients. This abstraction transforms a complicated problem into an intuitive formulation and sets the stage for delivering clinically implementable solutions in the future.

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VI Congreso de Jóvenes Investigadores

Real Sociedad Matemática Española

León, Febrero de 2023

A mathematical model for the Home Health Care Routing and Scheduling Problem.

Consuelo Parreño-Torres,[*] Miguel Reula,[†] Carlos Lamas-Fernandez,[‡]
Antonio Martinez-Sykora[§]

Home health care services are community health and nursing services that provide multiple coordinated care services to a patient in his or her home. Some of the most frequently performed tasks are medical supervision, support with household chores and assistance with personal hygiene. With an increasingly ageing population, the importance of home health care services is growing. Reducing the use and length of hospital stays of patients, as well as reducing the number of prolonged hospitalisations of patients, by trying to enable people to live at home for as long as possible, even at the end of their lives, has become a key issue for global health policies.

Home health care services are usually ordered by a doctor and then delivered through specialised teams that are typically integrated by a number of staff members with varying skills, starting locations and working hours. Patients require different types of care, during specific time windows, and might have special requirements, such as needing two staff members, or multiple visits with some sort of temporal dependency between them. This means that these teams need to decide which staff member will visit each patient, as well as the routes they will take to do so. Of course, these decisions have an impact on the total time required to see all patients, as well as the timeliness of the visits or the ability of maintain preferred staff-patient assignments. This kind of planning problem is known in the literature as the Home Health Care Routing and Scheduling Problem (HHCRSP), and has been extensively studied under different variants to model the requirements of different care providers. In this work, we present a novel Integer Linear Programming formulation for the HHCRSP which considers a wide range of practical aspects of the problem, and generalises previous models. The aim is fourfold: (i) to minimise the total daily distance travelled by care-givers, (ii) to minimise the total tardiness, avoiding unnecessary waiting times for patients, (iii) to balance delays between different patients, so as to provide a fair allocation of un-avoidable waiting times between them, (iv) as well as to minimise non-preferences between caregivers and patients.

We show that our formulation can be easily adapted to solve more specific variants published in the literature and that, using the same computing set-up, we are able to obtain more optimal solutions, feasible solutions and better lower bounds for benchmark instance sets.

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VI Congreso de Jóvenes Investigadores
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León, Febrero de 2023

A Branch and Price for the Min-Max Close-Enough Arc Routing Problem

Miguel Reula * Nicola Bianchessi Ángel Corberán Isaac Plana José María Sanchis

Arc routing problems are traditionally concerned with defining one or more routes that traverse a given set of arcs and/or edges which require to be served. However, throughout the years, many research studies have emerged enabling to go beyond this point of view. This is the case of the Close-Enough Arc Routing Problem (CEARP), which does not assume that customers are located on specific arcs, but that they can be served by traversing any arc of a given subset. This problem has real-life applications such as routing for automatic meter reading or inventory management. The idea is that if a device with a radio frequency receiver comes within a certain distance of a meter or tag, it collects its data. Therefore, only a few streets/aisles that are sufficiently close to the meters/tags need to be traversed. In this paper we introduce a generalization of this problem, the Min-Max Close-Enough Arc Routing Problem. In particular, we consider a CEARP in which a fleet of vehicles must serve the set of customers while trying to balance the length of the routes, i.e., minimize the length of the largest route.

In the work [1], we present two formulations of the problem; the first one is the traditional arc-based formulation making use of arc and servicing variables, and the second one a route-based set covering formulation. Based on the respective formulations, we proposed two exact methods to solve the problem, a branch-and-cut and a branch-and-price algorithm. Extensive computational experiments to compare the performance of the algorithms are carried out, and as it will be shown, the results demonstrate the efficiency of the proposed branch-and-price algorithm when the number of vehicles considered increases.

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VI Congreso de Jóvenes Investigadores

Real Sociedad Matemática Española

León, Febrero de 2023

Sustainable last-mile delivery using public transport

A. Santini[*] D. Delle Donne[†] C. Archetti[‡]

The boom of e-commerce in the last decade, and its growth projections for the coming years, raises new challenges as retailers and couriers innovate their supply chains to keep up with demand. Last-mile delivery (LMD), the segment of the supply chain which starts at the last distribution centre and ends at the customer's doorstep, is particularly affected. The growth of volume in LMD has caused concern, especially in dense urban environments where the externalities (traffic, emissions, etc.) have become noticeable. Several authors in Operational Research, Environmental Engineering, Urban Planning and Economics have proposed alternative LMD implementations which aim at reducing the externalities while guaranteeing a timely and inexpensive service. In this work, we focus on the practice of using spare capacity on public transport vehicles to move parcels into dense cities.

In particular, we tackle the problem of coordinating a three-echelon last-mile delivery system. In the first echelon, trucks transport parcels from distribution centres outside the city to public transport stops. In the second echelon, the parcels move on public transit and reach the city centre. Finally, in the third echelon a fleet of zero-emission vehicles picks up the parcel at public transport stops and delivers them to customers.

We prove that existing compact formulations of this problem cannot take into account all the required constraints, and introduce an extended formulation with two exponential set of variables (corresponding, respectively, with the first and third echelons). We propose a price-and-branch algorithm and compare several pricing algorithms on specially-constructed graphs.

To price the first-echelon variables we solve a resource-constrained elementary shortest path problem in which the cost of a label depends on the arrival time at a node. Pricing the third-echelon variables amounts to solving a sequence of "classical" resource-constrained elementary shortest path problems. The corresponding graphs are often so large that it is impossible to complete even just one round of pricing. Therefore, to obtain a primal bound we generate valid columns with a pricing heuristic. To obtain a dual bound, we explore several strategies: state-space relaxation, a Lagrangean-based bound and a decomposition-based bound. In particular, the decomposition-based bound disregards the synchronisation constraints imposed by the second echelon and bounding the cost of the first and third echelons separately. We obtain these bounds solving a Generalised Vehicle Routing Problem with Time Windows (for the first echelon) and a Multi-Depot Vehicle Routing Problem with Time Windows (for the third echelon).

Preliminary results confirm that our approach can obtain both high-quality primal solutions and tight dual bounds. In particular, we could find the optimum for all instances for which it is known, and we improved the best known solutions in most other instances from the literature.

Agradecimientos The work of Alberto Santini has received funding from the European Union's Horizon 2020 research and innovation programme under the Marie Skłodowska Curie grant agreement number 945380.

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VI Congreso de Jóvenes Investigadores
Real Sociedad Matemática Española
León, Febrero de 2023

El problema del cartero rural con una flota de drones

James F. Campbell,* Ángel Corberán,** Isaac Plana,*** José M. Sanchis,**** Paula Segura****

Los problemas de rutas por arcos (ARPs) consisten en encontrar una ruta, o un conjunto de rutas, de mínimo coste que atraviesen un conjunto de enlaces (arcos y/o aristas), llamados enlaces requeridos, en un grafo dado. Algunos problemas clásicos de rutas por arcos son el Problema de Cartero Chino, donde un único vehículo tiene que atravesar la totalidad de enlaces del grafo, el Problema del Cartero Rural, donde únicamente se necesita recorrer un subconjunto de enlaces de la red, o el Problema de Rutas por Arcos con Capacidades, donde un conjunto de vehículos con capacidad limitada deben recorrer conjuntamente los enlaces requeridos con coste mínimo. En los ARPs clásicos, las calles que necesitan un servicio de limpieza, las vías donde se debe eliminar la nieve o las tuberías que requieren ser inspeccionadas, por ejemplo, se representan mediante aristas o arcos de un grafo, siendo irrelevante la forma que estas vías tienen (aunque no su coste o distancia), ya que los vehículos terrestres deben atravesarlas de principio a fin (están obligados a seguir los enlaces de la red). La utilización de drones (vehículos aéreos no tripulados) para optimizar el servicio de los enlaces requeridos presenta nuevas oportunidades a la hora de abordar estos problemas: los drones pueden viajar fuera de la red y servir únicamente una parte de un enlace requerido, utilizando varias rutas para servir completamente dicho enlace.

En esta charla abordaremos el problema del cartero rural con K drones con restricción de longitud (LC K–DRPP). Se trata de un problema de optimización continua donde un conjunto de líneas rectas o curvas de una red deben ser atravesadas (servidas) con coste mínimo por una flota de drones homogéneos. Dado que el rango de vuelo del drone es limitado, la longitud de cada ruta está limitada por un valor L. Los drones no están restringidos a viajar sobre la red, y pueden entrar y salir de cada línea en cualquiera de sus puntos, sirviendo sólo una porción de la misma [1]. Para hacer frente a este problema, las instancias del LC K–DRPP pueden ser discretizadas aproximando cada línea mediante una cadena poligonal con un conjunto finito de puntos, y permitiendo a los drones entrar y salir de cada línea únicamente en esos puntos. Así, se obtiene una instancia de un problema del cartero rural para K vehículos (LC K–RPP), que es un problema discreto de rutas que puede resolverse con técnicas de optimización combinatoria. Sin embargo, cuando el número de puntos es demasiado grande, la instancia puede resultar difícil de resolver incluso para algoritmos heurísticos, por lo que resulta necesaria la implementación de un procedimiento que elija un conjunto pequeño pero significante de puntos y segmentos con los que aproximar cada línea. Presentaremos aquí una formulación para el LC K–RPP, varias familias de desigualdades válidas para el problema que inducen faceta del poliedro de soluciones y la implementación de un algoritmo de ramificación y corte que incorpora procedimientos de separación para dichas desigualdades, siendo este algoritmo la rutina principal de un método iterativo para encontrar soluciones para el LC K–DRPP [2]. Acompañaremos este estudio con algunos resultados computacionales.

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VI Congreso de Jóvenes Investigadores

Real Sociedad Matemática Española

León, Febrero de 2023

A Mathematical Programming approach to the Minimum Normalized Cut Problem

A. Diego Ponce*, B. Justo Puerto*, C. Francisco Temprano*

The normalized cut function [1] was defined to solve some issues concerning the interpretability of the minimum cut problem, which is a classical problem in graph theory whose aim is to provide the bipartition that minimizes the number of edges between nodes from different subsets, applied to community detection problems.

Instead of considering just the number of external edges of each community, the minimum k-way normalized cut problem tries to minimize the external edge density of each community of a k-partition [3], also considering the number of internal edges.

Some mixed integer linear programming formulations and a Branch-and-Price algorithm [2] are developed, and their performances are compared over some families of random instances.

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VI Congreso de Jóvenes Investigadores
Real Sociedad Matemática Española
León, Febrero de 2023

Improving algorithm efficiency for fairness allocation in optimization

A. Alberto Torrejón Valenzuela *   B. Miguel A. Pozo Montaño*   C. Justo Puerto Albandoz*

Ordered median optimization has been proven to be a powerful tool to model many features in well-known problems from the literature (location problems, multiobjective optimization, complex networks design, etc.). Considering ordering procedures straightforward in compact formulations allows us to extend the range of application of the problems, including the modeling of preferences, equity, avoiding envy, unhappiness, among other characteristics of interest.

In this talk we present new enhancements for the Discrete Ordered Median Problem (DOMP), a location problem where client-facility allocations are ranked (see, e.g., [1]). We review the current most competitive formulations in the literature based on properties of the k-sum optimization (see [2]) and present a series of Benders decomposition approaches for DOMP.

Considering this approaches, state-of-the-art results have been obtained for different ordered weighting vectors. The outcome is that a clever implementation of the Benders decomposition is very effective, as its performance is comparable and sometimes even improved the most competent and sophisticated formulations and algorithms proposed in the literature allowing enhancements in terms of scalability.

Acknowledgements: This research was supported by the project *"Nuevos resultados sobre los problemas de diseño y optimización en redes complejas: Aplicaciones al diseño de ciudades inteligentes"* (FEDER-US-1256951). This support is gratefully acknowledged.

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VI Congreso de Jóvenes Investigadores

Real Sociedad Matemática Española

León, Febrero de 2023

The Hampered Traveling Salesman Problem with Neighbourhoods

J. Puerto,* C. Valverde†

Routing problems are considered classical problems in the core of combinatorial optimisation. Among them, the Traveling Salesman Problem (TSP) is one of its more important representatives and it has been studied extensively in many different forms. The problem is well-known to be NP-hard. Its analysis has led in the last decades to methodological advances and new optimisation techniques that have allowed solving real life instances that few years ago were beyond solvable limits. The TSP has been studied in its geometric and pure combinatorial forms because of its algorithmic and practical implications. One very interesting extension arises when we require the tour to visit a set of regions rather than points. This version of the problem is called the TSP with Neighbours (see [1]) which is APX-hard to approximate even for regions that are (intersecting) line segments. They can represent regions that the drone must reach and where the customers are willing to pick up the orders (they can be seen as uniform probability densities) in the delivery industry. Moreover, they can be also used for modelling some areas that must be inspected by the drone (whenever visiting a point of these areas is enough to consider them as inspected). On the other hand, the graphic TSP (see [2]), where distances are measures by geodesics (shortest paths in the respective metric space), has also attracted the attention of many researchers since it models, in a natural way, TSP routes among forbidden barriers, robot motion planning and automatic navigation in computer games.

This talk deals with two different route design problems in a continuous space with neighbours and barriers: the shortest path and the traveling salesman problems with neighbours and barriers. Each one of these two elements, neighbours and barriers, make the problems harder than their standard counterparts. Therefore, mixing both together results in a new challenging problem that, as far as we know, has not been addressed before but that has applications for inspection and surveillance activities and the delivery industry assuming uniformly distributed demand in some regions. Exact mathematical programming formulations for both problems are provided assuming polygonal barriers and neighbours that are second-order cone (SOC) representable. These hypotheses give rise to mixed integer SOC formulations that are preprocessed and strengthened with valid inequalities. The work also reports computational experiments showing that our exact method can solve instances with 80 neighbourhoods and a range between 125-145 barriers.

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Matemáticas en la Sociedad de la Información

Organizadores: Sara Díaz, Irene Márquez, Verónica Requena y Carlos Vela

Jueves 9 de Febrero
Moderadora: *Verónica Requena, Carlos Vela y Miguel Ángel Navarro*

10:00 *Self-dual Convolutional Codes*, Julia Lieb

10:30 *Nuevas ideas para mejorar la seguridad de un criptosistema de tipo McEliece basado en códigos convolucionales*, Miguel Beltrá

11:30 *Bounds for flag codes*, Miguel Ángel Navarro Pérez

12:00 *Subfield subcodes of projective Reed-Muller Codes*, Rodrigo San José Rubio

12:30 *Permutation Decoding in Group Codes*, Fabián Ricardo Molina Gómez

13:00 *Quantum codes defined by trace-depending polynomials*, Helena Martín Cruz

17:00 *On the possible sequences of generalized weights of linear codes*, Umberto Martínez Peñas

17:30 *Gray maps and linearities*, Carlos Vela

18:00 *Simetrías gauge para preservación de la privacidad aprendizaje automático*, Alex Pozas-Kerstjens

18:30 *Rank metric codes over Galois rings*, Markel Epelde

Viernes 10 de Febrero
Moderadores: *Daniel Sadornil y Miguel Beltrá*

10:00 *Functional Commitments for Circuits from Falsifiable Assumptions*, David Balbás

10:30 *Generación eficiente de números con propiedades de ruido gaussiano*, Francisco Javier Soto Sánchez

11:30 *A generalized Euclidean algorithm for the multisequence Skew-Feedback Shift-Register synthesis problem*, José Manuel Muñoz

12:00 *La transformación de Fujisaki-Okamoto y su relevancia en algoritmos post-cuánticos*, Miguel Ángel González de la Torre

12:30 *Design of a Post-Quantum Group Authenticated Key Exchange protocol using Classic McEliece, Kyber, NTRU, and Saber*, Misael Enrique Marriaga Castillo

VI Congreso de Jóvenes Investigadores

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León, Febrero de 2023

Nuevas ideas para mejorar la seguridad de un criptosistema de tipo McEliece basado en códigos convolucionales.

P. Almeida, Miguel Beltrá [*], Diego Napp

En este trabajo presentamos algunas ideas nuevas para construir un criptosistema de McEliece utilizando códigos convolucionales. El principal problema del criptosistema de McEliece es el tamaño de su clave pública, considerablemente más grande que en otros criptosistemas de clave pública. Esto se debe a que la clave pública viene dada por la matriz generadora de un código lineal. La idea que se propone es enmascarar la matriz generadora de un código de Reed-Solomon generalizado, conocidos por ser códigos de máxima distancia de separación y tener una buena capacidad correctora y un algoritmo de decodificación eficiente, con matrices polinomiales que nos permitan esconder la estructura del código. El mensaje transmitido es una secuencia de bloques que se mezclan entre si lo suficiente como para que un atacante no pueda recuperar bloques sueltos, teniendo que tratar la secuencia entera como un único bloque y aplicar ataques conocidos a una matriz con un tamaño considerablemente grande. Además se realiza un análisis de los ataques básicos habituales a este tipo de criptosistemas teniendo en cuenta estos factores.

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VI Congreso de Jóvenes Investigadores
Real Sociedad Matemática Española
León, Febrero de 2023

Bounds for flag codes

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Let q be a prime power and $\mathbb{F}_q$ be the finite field with q elements. For every $n \geq 2$, we will write $\mathbb{F}_q^n$ to denote the n-dimensional vector space over $\mathbb{F}_q$. Given positive integers $t_1 < \ldots < t_r < n$, a flag $\mathcal{F}$ of type $\mathbf{t} = (t_1, \ldots, t_r)$ on $\mathbb{F}_q^n$ is a sequence $\mathcal{F} = (\mathcal{F}_1, \ldots, \mathcal{F}_r)$ of $\mathbb{F}_q$-subspaces of $\mathbb{F}_q^n$ such that $\mathcal{F}_1 \subset \ldots \subset \mathcal{F}_r$ and $\dim(\mathcal{F}_i) = t_i$, for every $1 \leq i \leq r$. Flag codes were introduced in [4] as nonempty sets of flags of a specific type $\mathbf{t}$ on $\mathbb{F}_q^n$.

Given two flags $\mathcal{F}, \mathcal{F}'$ of type $\mathbf{t}$, their flag distance is the value $d_f(\mathcal{F}, \mathcal{F}') = \sum_{i=1}^{r} d_S(\mathcal{F}_i, \mathcal{F}'_i)$, where $d_S(\mathcal{F}_i, \mathcal{F}'_i) = 2(t_i - \dim(\mathcal{F}_i \cap \mathcal{F}'_i))$, and the minimum distance of a flag code $\mathcal{C}$ is $d_f(\mathcal{C}) = \min\{d_f(\mathcal{F}, \mathcal{F}'),\ \mathcal{F}, \mathcal{F}' \in \mathcal{C},\ \mathcal{F} \neq \mathcal{F}'\}$.

One of the main problems in Coding Theory is determining the maximum possible cardinality for codes with a prescribed minimum distance. In our setting, we write $A_q^f(n, \mathbf{t}, d)$ to denote the size of the largest flag code of type $\mathbf{t}$ on $\mathbb{F}_q^n$ and having minimum distance d. In [1], we can find the first bounds for $A_q^f(n, \mathbf{t}, d)$ for specific choices of the type $\mathbf{t}$ and for the maximum possible minimum distance. Later, in [3], the author obtains bounds for $A_q^f(n, \mathbf{t}, d)$ for different values of the distance and $\mathbf{t} = (1, \ldots, n-1)$.

In our work [2], upper bounds for $A_q^f(n, \mathbf{t}, d)$ for every type vector are computed. Notice that any value of the flag distance defined as above might be obtained as the sum of different combinations of subspace distances. For this reason, before studying the value $A_q^f(n, \mathbf{t}, d)$, we analyze in which ways a given flag distance value d can be attained. This idea gives rise to the notion of distance vector. More precisely, the distance vector between two flags $\mathcal{F}$ and $\mathcal{F}'$ is the sequence

$$\mathbf{d}(\mathcal{F}, \mathcal{F}') = (d_S(\mathcal{F}_1, \mathcal{F}'_1), \ldots, d_S(\mathcal{F}_r, \mathcal{F}'_r))$$

and it represents a feasible realization of the distance value $d_f(\mathcal{F}, \mathcal{F}')$.

In this talk, we present and characterize the concept of distance vector. Moreover, we use the information provided by this new object and derive new bounds for the cardinality of flag codes for every choice of the parameters.

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VI Congreso de Jóvenes Investigadores

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León, Febrero de 2023

Functional Commitments for Circuits from Falsifiable Assumptions

David Balbás[*†] Dario Catalano[‡] Dario Fiore[*] Russell W. F. Lai[§]

A functional commitment (FC) scheme [1] allows one to commit to a vector $\vec{x}$ and later produce a short opening proof of $(f, f(\vec{x}))$ for any admissible function f. FC schemes can be seen as a generalisation of vector and polynomial commitment schemes, increasingly popular in the last years. Their security, called evaluation binding, ensures that it is hard to open the commitment to the same function f and different outputs $y \neq y'$. Unlike succinct non-interactive arguments (SNARG) which provide a stronger soundness guarantee but typically require non-falsifiable assumptions, the evaluation binding of FC schemes can often be based on falsifiable assumptions and is sufficient for certain applications such as constructing homomorphic signatures (HS). Since their inception, FC schemes supporting ever more expressive classes of functions have been proposed, with the state-of-the-art supporting low-degree polynomial maps [2].

In this work [3], we present the first functional commitment scheme for arithmetic circuits, based on either pairing-based or lattice-based falsifiable assumptions. Our FCs require to fix a-priori only the maximal width of the circuit to be evaluated, and have opening proofs whose size only depends on the depth of the circuit. As a direct consequence, we obtain the first pairing- and lattice-based constructions of HS for unbounded depth circuits. Prior to this work, the only HS for general circuits is lattice-based and requires bounding the circuit depth at setup time.

Acknowledgements This work has received funding in part from the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation program under project PICOCRYPT (grant agreement No. 101001283), by the Spanish Government under projects SCUM (ref. RTI2018-102043-B-I00), and RED2018-102321-T, by the Madrid Regional Government under project BLOQUES (ref. S2018/TCS-4339), and by a research grant from Nomadic Labs and the Tezos Foundation.

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Rank metric codes over Galois rings

Markel Epelde*

In 1985, Gabidulin introduced the rank metric in coding theory over finite fields [3], and used this kind of of codes in a McEliece cryptosystem, six years later [2]. In this talk, we consider rank metric codes over Galois rings. We propose a suitable metric for codes over such rings, and show its main properties. With this metric, we define Gabidulin codes over Galois rings, propose an efficient decoding algorithm for them, and hint their cryptographic application.

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Real Sociedad Matemática Española

León, Febrero de 2023

Quantum codes defined by trace-depending polynomials

C. Galindo[*] F. Hernando[*] H. Martín-Cruz[*] D. Ruano[†]

The existence of polynomial time algorithms for prime factorization and discrete logarithms on quantum computers is a clear example that illustrates the importance of quantum computing [4]. Quantum error-correcting codes are essential for this type of computation since they protect quantum information from decoherence and quantum noise. Many of the known quantum error-correcting codes are stabilizer codes. A *stabilizer code* $\mathcal{C} \neq \{0\}$ is the common eigenspace (with respect to the eigenvalue 1) of a commutative subgroup of the error group generated by a suitable error basis on the space $\mathbb{C}^{q^n}$, where $\mathbb{C}$ denotes the complex field, q is a prime power and n is a positive integer. A main advantage of stabilizer quantum error-correcting codes is that they can be constructed from self-orthogonal classical linear codes [3].

Quantum error-correcting codes with good parameters can be constructed from evaluation codes by evaluating polynomials at the roots of the polynomial trace $\mathrm{tr}_{2n}(X) = X + X^q + \cdots + X^{q^{2n-1}} \in \mathbb{F}_{q^{2n}}[X]$ [1] . In this talk, we propose to evaluate polynomials at the roots of trace-depending polynomials $a + \mathrm{tr}_{2n}(h(X))$, where $a \in \mathbb{F}_{q^{2n}}$ and $h(X) \in \mathbb{F}_{q^{2n}}[X]$, and show that this procedure gives rise to stabilizer quantum error-correcting codes with a wider range of lengths than in [1] and with excellent parameters. Namely, we are able to provide new binary records according to [2] and non-binary codes improving the ones available in the literature.

Acknowledgements Partially funded by MCIN/AEI/10.130
39/501100011033 and by "ERDF A way of making Europe", grants PGC2018-096446-B-C21 and PGC2018-096446-B-C22, by MCIN/AEI/10.13039/501100011033 and by "ESF Investing in your future", grant RYC-2016-20208, as well as by Universitat Jaume I, grants UJI-B2021-02 and PREDOC/2020/39.

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VI Congreso de Jóvenes Investigadores
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León, Febrero de 2023

Subfield subcodes of projective Reed-Muller codes

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The family of Reed-Solomon codes has excellent parameters, but at the cost of having to be defined over big finite fields in order to have an appropriate length for applications. A way to obtain good parameters over a smaller finite field is to consider subfield subcodes. Let q be a power of a prime. We consider the field extension $\mathbb{F}_{q^s} \supset \mathbb{F}_q$. Given a code $C \subset \mathbb{F}_{q^s}^n$, the subfield subcode with respect to the previous field extension is defined as $C^\sigma := C \cap \mathbb{F}_q^n$. These subcodes have been studied for different families of codes, being one of the more general ones the family of J-affine variety codes [2]. For this family, which contains Reed-Solomon codes and Reed-Muller codes, an explicit basis can be obtained, which also gives the dimension of the code.

The interest of these subcodes is that in some cases this construction gives codes with parameters bounded by the parameters of the original code, but in a smaller field. In fact, in some cases, the parameters of these codes are the best known parameters for the corresponding field size [3]. This construction is also useful for applications because some properties of the original code are preserved for the subfield subcodes [1].

In our work, we are interested in evaluation codes over the projective space instead of the affine space (as in [2]). The best known families of such codes are the family of projective Reed-Solomon codes (also known as doubly extended Reed-Solomon codes) and the family of projective Reed-Muller codes. The aim of our work is to obtain a set of polynomials such that its image by the evaluation map forms a basis for the subfield subcode, similarly to the affine case [1].

For the family of projective Reed-Solomon codes we are able to obtain a base for both the subfield subcodes and their dual codes, and we are also able to do the same for projective Reed-Muller codes over the projective plane $\mathbb{P}^2$. The necessary tools in order to deal with the general case of projective Reed-Muller codes over $\mathbb{P}^m$ are also developed.

Acknowledgments This work was supported in part by the following grants: Grant PID2019-104844GB-I00 funded by MCIN/AEI/10.13039/501100011033, Grant PGC2018-096446-B-C21 funded by MCIN/AEI/10.13039/501100011033 and "ERDF A way of making Europe", Grant TED2021-130358B-I00 funded by MCIN/AEI/10.13039/501100011033 and by the "European Union NextGenerationEU/PRTR", Grant RYC-2016-20208 funded by MCIN/AEI/10.13039/501100011033 and "ESF Investing in your future", and Grant FPU20/01311 funded by the Spanish Ministry of Universities.

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VI Congreso de Jóvenes Investigadores

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León, Febrero de 2023

La transformación de Fujisaki-Okamoto y su relevancia en algoritmos post-cuánticos

Miguel Ángel González de la Torre *   Luis Hernández Encinas†

La criptagrafía post-cuántica es una de las áreas con mayor desarrollo en los últimos años. El proceso de estandarización del NIST ha sido un foco central en el desarrollo de algoritmos post-cuánticos. Esta comunicación, basandose en dos trabajos previos [3], [4], tiene como proposito presentar la estructura utilizada en la categoría PKE/KEM para hacer los algoritmos seguros, denominada Transkformación de Fujisaki-Okamoto (FO).

La primera versión de la transformación FO se publicón en 1999 y consistía en la estructura de un criptosistema híbrido, tal que el criptosistema resultante alcanza una noción de seguridad más elevada que las consideradas incialmente. Las versiones de la transformación FO posteriores [1], [2], [5] se centran en los algoritmos de clave pública y se han estudiado distintas variantes, dependiendo del diseño o las hipótesis consideradas. Las nuevas versiones de la transformación FO consisten en convertir un PKE en un KEM, que además alcanza una noción de seguridad mayor. Estas transformaciones son las que utilizan los algoritmos post-cuánticos y mediante las cuales los KEM resultantes obtienen la noción de seguridad semántica IND-CCA, establecida como referencia por el NIST.

Agradecimientos This work was supported in part by ORACLE Project, with reference PCI2020-120691-2, funded by MCIN/AEI/10.13039/501100011033, and European Union “NextGenerationEU/PRTR”, in part by the Spanish State Research Agency (AEI) of the Ministry of Science and Innovation (MCIN), project P^{2}QProMeTe (PID2020-112586RB-I00/AEI/10.13039/501100011033), and in part by the EU Horizon 2020 research and innovation programme, project SPIRS (Grant Agreement No. 952622).

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On the possible sequences of generalized weights of linear codes

E. Gorla [*], E. Lorenzo García[*] U. Martínez-Peñas [**] F. Salizzoni[*]

In this work, we explore what sequences of positive integers are the sequence of generalized weights of a linear code, allowing the underlying field size to be large enough. We will consider the sum-rank metric and its particular case, the Hamming metric.

For a positive integer r, let $[r] = \{1, 2, \dots, r\}$. For positive integers $m \geq n$ and a prime power q, we denote by $\mathbb{F}_q^{m\times n}$ the set of $m \times n$ matrices with entries in the field $\mathbb{F}_q$. We denote by $\mathbb{M}$ the $\mathbb{F}_q$-linear vector space

$$\mathbb{M} = \mathbb{F}_q^{m_1\times n_1} \times \dots \times \mathbb{F}_q^{m_t\times n_t},$$

where $t, n_1, \dots, n_t, m_1, \dots, m_t$ are positive integers such that $m_1 \geq \dots \geq m_t$ and $m_i \geq n_i$ for $i \in [t]$. We write $n = n_1 + \dots + n_t$. For $C = (C_1, \dots, C_t) \in \mathbb{M}$ we define the sum-rank weight of C as $\mathrm{wt}_{SR}(C) = \sum_{i=1}^{t} \mathrm{Rk}(C_i)$. We also define the minimum (sum-rank) distance of a code $\{0\} \neq \mathcal{C} \subseteq \mathbb{M}$ as $\mathrm{d}(\mathcal{C}) = \text{mín}\{\mathrm{wt}_{SR}(C) : C \in \mathcal{C} \setminus \{0\}\}$.

The minimum distance of a code can be generalized to what is known as generalized weights $\mathrm{d}_r(\mathcal{C})$, for $r \in [\dim(\mathcal{C})]$. Such numbers, introduced in [1], form a non-decreasing sequence such that $\mathrm{d}_1(\mathcal{C}) = \mathrm{d}(\mathcal{C})$. When $m_1 = m_2 = \dots = m_t = 1$, then

$$\mathrm{d}_r(\mathcal{C}) = \text{mín}\{|\mathrm{Supp}(\mathcal{D})| : \mathcal{D} \subseteq \mathcal{C} \text{ lineal and } \dim(\mathcal{D}) = r\}$$

is the rth generalized Hamming weight of $\mathcal{C}$, where $\mathrm{Supp}(\mathcal{D}) = \{i \in [n] : \exists D \in \mathcal{D}, D_i \neq 0\}$.

Our main results are the following.

Theorem 1. *Assume that $m_1 = m_2 = \dots = m_t = 1$ (that is, we are in the Hamming-metric case) and $q \geq n - 1$. For any integer $k \in [n]$ and any sequence $1 \leq d_1 < d_2 < \dots < d_k \leq n$, there is a linear code $\mathcal{C} \subseteq \mathbb{F}_q^n$ such that $\mathrm{d}_r(\mathcal{C}) = d_r$, for $r \in [k]$.*

Theorem 2. *In the general case $1 \leq m_t \leq \dots \leq m_1$, $n_i \leq m_i$ for $i \in [t]$, and $1 \leq d \leq n_1 + \dots + n_t$. Let $\mathcal{D} \subseteq \mathbb{F}_q^{m_1\times n_1} \times \dots \times \mathbb{F}_q^{m_t\times n_t}$ be an MSRD code [1] of minimum distance d and assume that there exists a chain of MSRD codes $\mathcal{D} = \mathcal{D}_d \supseteq \mathcal{D}_{d+1} \supseteq \dots \supseteq \mathcal{D}_n$ such that $\mathrm{d}(\mathcal{D}_h) = h$, for $h = d, d+1, \dots, n$. A sequence of positive integers $d_1, d_2, \dots, d_k$ is the sequence of generalized sum-rank weights of a linear subcode $\mathcal{C} \subseteq \mathcal{D}$ if and only if $k \leq \dim(\mathcal{D})$ and $d_1, \dots, d_k$ is a subsequence of the sequence of generalized weights of $\mathcal{D}$.*

Agradecimientos The author gratefully acknowledges the support from a María Zambrano contract by the University of Valladolid, Spain (Contract no. E-47-2022-0001486).

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VI Congreso de Jóvenes Investigadores

Real Sociedad Matemática Española

León, Febrero de 2023

Self-dual Convolutional Codes

Julia Lieb*

Convolutional codes form a class of error-correcting codes which is mainly used for sequential encoding and decoding of a stream of data. They are a generalization of the classical block codes to the polynomial setting, i.e. an (n, k) convolutional code $\mathcal{C}$ is defined as $\mathbb{F}[z]$-submodule of $\mathbb{F}[z]^n$ of rank k. The dual of a convolutional codes $\mathcal{C}$ is defined as

$$\mathcal{C}^{\perp} = \{ \ v \in \mathbb{F}[z]^n \mid \ < v, c >= 0 \ \ \forall c \in \mathcal{C} \ \} \, .$$

For block codes, the subclass of self-dual codes has been widely investigated, whereas there is not much known about self-dual convolutional codes in the general case. In this talk, we will introduce self-dual convolutional codes, give criteria to check whether a convolutional code is self-dual as well as some basic properties of self-dual convolutional codes. Furthermore, we will present two construction methods which allow to build self-dual convolutional codes from other self-dual convolutional codes of shorter length and investigate some properties of these constructions. Finally, we give a classification of all self-dual convolutional codes of rate $\frac{1}{2}$ and of rate $\frac{2}{4}$.

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Design of a Post-Quantum Group Authenticated Key Exchange protocol using Classic McEliece, Kyber, NTRU, and Saber

José Ignacio Escribano Pablos,[*] Misael Enrique Marriaga[†] Ángel Luis Pérez del Pozo[‡]

Group authenticated key exchange protocols (GAKE) are cryptographic tools enabling a group of several users communicating through an insecure channel to securely establish a common shared high-entropy key. In the last years, the need to design cryptographic tools which provide security in the presence of attackers with access to quantum resources has become unquestionable; the field dealing with these types of protocols is usually referred to as *Post-Quantum Cryptography*. The U.S. National Institute for Standards and Technology (NIST) launched in 2017 an open call to find suitable post-quantum public-key algorithms for standardization. In this work, we design a GAKE that can be instantiated with any key encapsulation mechanism (KEM) that satisfies the strong security notion IND-CCA, matching NIST's requirements for this primitive.

Acknowledgements The authors would like to thank María I. González Vasco for her help during the proccess of ellaboration of the manuscript, through fruitful discussions and useful comments and suggestions.

This research was sponsored in part by the NATO Science for Peace and Security Programme under grant G5448 and in part by Spanish MINECO under grant PID2019-109379RB-I00.

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VI Congreso de Jóvenes Investigadores

Real Sociedad Matemática Española

León, Febrero de 2023

Permutation Decoding in Group Codes

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Group codes can be identified with (two-sided) ideals of a group algebra. Here, in the case of a semisimple group algebra $\mathbb{K}G$, we study a permutation decoding algorithm in group codes. We use the decomposition of $\mathbb{K}G$ as the sum of two ideals (one of them the group code) generated by central idempotents to obtain results that allow a justification for the implementation of this algorithm. We also present some cases where possible PD$-$sets can be considered to make it more reliable in error correction.

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VI Congreso de Jóvenes Investigadores

Real Sociedad Matemática Española

León, Febrero de 2023

A generalized Euclidean algorithm for the multisequence Skew-Feedback Shift-Register synthesis problem*

J.M. Muñoz† F.J. Lobillo†

Skew cyclic codes, which are defined as left ideals in a quotient ring from a skew polynomial ring, can be used as nearest-neighbor error-correcting codes. This can be done by carefully choosing the defining set of the code, so that a lower bound for its minimum Hamming distance is guaranteed. Three of these bounds are the BCH bound, the Hartmann-Tzeng (HT) bound, and the Roos bound (see [1, 4]); the last two ones being a general case of the first one, and therefore possibly closer to the actual distance. Additionally, the defining set generates a pattern in the syndromes of a corrupted codeword which, when found, reveals the corrupted positions. After finding those, decoding is done by solving a linear equation system.

The pattern to be found is the shortest skew-feedback shift register (SFSR) which, when initialized with the first items in one of multiple sequences of syndromes, iteratively generates the rest of the sequence. This means solving the multisequence SFSR synthesis problem. Decoding up to the BCH bound requires solving the single-sequence version of the problem, which can be done using the extended Euclidean algorithm for skew polynomials, as done in [3].

These constructions also work for (non-skew) cyclic codes, where the analogous multisequence linear feedback shift register (LFSR) synthesis problem can be solved, as shown in [2], by generalizing the extended Euclidean algorithm for polynomials. This results in nearest-neighbor decoding up to the HT and Roos bounds for cyclic codes. Our work replicates this generalization of the extended Euclidean algorithm for skew polynomials, so that the same can be achieved for skew cyclic codes. This will be a step towards generalizing the work in [3].

In order to solve the multisequence SFSR synthesis problem, the $m > 1$ input sequences of length n are first arranged as a single skew polynomial $S(x)$ in such a way that, for some suitably chosen $M \geq m$ such that x^M is a central skew polynomial, the degree of $\lambda(x^M)S(x) \mod x^{Mn}$ is less than ℓM if and only if the SFSR of length ℓ formed by the coefficients of $\lambda(x^M) = \sum_{i=0}^{\ell} \lambda_i x^{iM}$ generates the input sequences. Then, such $\lambda(x^M)$ can be found by generalizing the extended Euclidean algorithm into one that computes, for each step, a Bézout-like identity

$$r(x) = u(x^M)S(x) + v(x)x^{Mn},$$

and returns $u(x^M)$ when $\deg r(x) < \deg u(x^M)$. This generalization requires, in turn, generalizing the right division algorithm.

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*Supported by grants PRE2020-09325 from MCIN/AEI / 10.13039/501100011033 and FSE “El FSE invierte en tu futuro” and PID2019-110525GB-I00 from Agencia Estatal de Investigación (AEI / 10.13039/501100011033)

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VI Congreso de Jóvenes Investigadores

Real Sociedad Matemática Española

León, Febrero de 2023

Gauge symmetries for privacy-preserving machine learning

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Vast amounts of data are routinely processed in machine learning pipelines, every time covering more aspects of our interactions with the world. However, the race for ever-better performing models is leaving other important aspects behind, like privacy. For instance, when the models processing the data are made public, is the safety of the data used for training it guaranteed? This is a question of utmost importance when processing sensitive data such as medical records.

In this talk I will argue and practically illustrate that fundamental mathematical insights can help in devising better privacy-preserving machine learning algorithms. First, I will show that standard neural networks are vulnerable to a type of privacy leak that involves global properties of the data used for training [1]. This is especially worrying because standard protection mechanisms [2] only focus in protecting information about the presence of concrete datapoints in the training dataset. Second, I will show that the concept of gauge symmetry is the key that allows to rigorously prove robustness against such vulnerability. Moreover, I will show that a specific family of architectures, called tensor networks [3], satisfy the required properties and this are good candidates for machine learning architectures. Interestingly, these architectures were developed within the community of physics for characterizing quantum many-body systems, thus demonstrating a pretty connection between mathematics, quantum physics and machine learning.

Agradecimientos This work is supported by the European Union (Horizon 2020 research and innovation programme-grant agreement No. 648913 and ERDF “A way of making Europe”), the Spanish Ministry of Science and Innovation (“Severo Ochoa Programme for Centres of Excellence in R&D” CEX2019-000904-S and ICMAT Severo Ochoa project SEV-2015-0554, and grants CEX2019-000904-S-20-4, MTM2014-54240-P, MTM2017-88385-P, PGC2018-098321-B-I00 and PID2020-113523GB-I00), the Spanish Ministry of Economic Affairs and Digital Transformation (project QUANTUM ENIA, as part of the Recovery, Transformation and Resilience Plan, funded by EU program NextGenerationEU), Comunidad de Madrid (PEJ-2021-AI/TIC-23267 and QUITEMAD-CM P2018/TCS-4342), and the CSIC Quantum Technologies Platform PTI-001.

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VI Congreso de Jóvenes Investigadores

Real Sociedad Matemática Española

León, Febrero de 2023

Generación eficiente de números con propiedades de ruido gaussiano

Francisco Javier Soto Sánchez   Ana Isabel Gómez[*]   Domingo Gómez-Pérez [**]
Andrew Tirkel [***]

La generación de ruido aleatorio es necesaria para la calibración de diversos instrumentos de medida presentes en satélites [3] y otras aplicaciones de comunicaciones. Las propiedades de la fuentes de ruido son críticas para el correcto rendimiento del instrumento, pero las implementaciones basadas en fenómenos físicos presentan desventajas en su utilización en casos prácticos como: no reproducibilidad, dependencia de las condiciones ambientales, etc. El uso de secuencias aleatorias para la producción de una fuente de ruido fiable en un acelerador hardware es una opción atractiva. Aunque existe una gran cantidad de literatura estudiando la teoría de la generación de muestras con distribución fija a partir de variables aleatorias uniformes [2], las implementaciones concretas están limitadas por la dificultad intrínseca del hardware [4].

En este trabajo hemos estudiado las posibles modificaciones de los generadores de números pseudoaleatorios clásicos para su adaptación a secuencias de números pseudoaleatorios con distribución Gaussiana. Partiendo de las propiedades de estos generadores, obtenemos los primeros resultados teóricos respecto a dos test estadísticos: el test de Shapiro-Wilks y la precisión en la cola de la distribución. Los primeros análisis computacionales muestran resultados prometedores con una implementación eficiente.

Agradecimientos Esta publicación es parte del Proyecto PID2019-110633GB-I00 financiado por MCIN/ AEI /10.13039/501100011033.

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Real Sociedad Matemática Española

León, Febrero de 2023

Gray maps and linearities

C. Fernández-Córdoba,* C. Vela,† M. Villanueva*

Since its first usage for solving a coding theory problem in 1994 the Gray maps have been used to make constructions of well-known families of linear and nonlinear codes. The Gray map was first presented as an isometry between $\mathbb{Z}_4$ and $\mathbb{Z}_2^2$ with the Lee and Hamming metrics respectively. Later, generalisation up to finite chain rings of such Gray map were defined. In this talk we will see some properties of this Gray map and how can be used to obtain a compact representation of some families of nonlinear codes [1, 2, 3, 4].

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Inferencia estadística frecuentista y aplicaciones

Organizadores: José Ameijeiras

Miércoles 8 de Febrero
Moderador: *José Ameijeiras Alonso*

10:00 *Métodos no paramétricos para comparar curvas ROC con covariables*, Arís Fanjul Hevia

10:20 *La correlación de distancias. Aplicación en Análisis de Supervivencia*, María Vidal García

10:40 *Un nuevo contraste de multimodalidad para datos circulares*, Diego Bolón Rodríguez

Jueves 9 de Febrero
Moderadora: *María Alonso Pena*

10:00 *Probability of default estimation using mixture cure models*, Rebeca Peláez Suárez

10:20 *Testing a linear-circular parametric regression model*, Andrea Meilán Vila

10:40 *The FMM approach for mathematical modelling of the 24-h cardiac rhythm*, Yolanda Larriba

Viernes 10 de Febrero
Moderadora: *Yolanda Larriba*

10:00 *A broad approach for kernel regression with circular covariates*, María Alonso Pena

10:20 *Survival analysis with a cure fraction and multidimensional covariates*, Beatriz Piñeiro Lamas

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Real Sociedad Matemática Española
León, Febrero de 2023

A broad approach for kernel regression with circular covariates

M. Alonso-Pena,[*] I. Gijbels[†] R.M. Crujeiras[‡]

Circular variables are those who present a periodic behavior, which makes classical statistical tools not suitable to perform inference for this type of variables (see [1] and [2] for an introduction to circular statistics). One example where different statistical methods are required is regression problems where one of the variables involved is circular (see, e.g. [3]).

In this work, based on [4] we focus on a general regression problem, where the response variable is a general real-valued variable (continuous or discrete) and the covariate presents a circular structure. We estimate a conditional characteristic nonparametrically, by maximizing the circular local likelihood . In addiiton, we derive a general rule for selecting the smoothing parameter and study some of its properties. Simulation experiments validate the good performance of the new methodology compared to other existing methods for selecting the parameters.

Finally, the methodology presented in this work can be applied to many practical contexts, such as meteorology, psychology, animal escapology or neurology.

Agradecimientos The authors gratefully acknowledge support from project C16/20/002 of the Research Fund KU Leuven, Belgium; project PID2020-116587GB-I00, funded by MCIN/AEI/10.13039 /501100011033; and the Competitive Reference Groups 2021–2024 (ED431C 2021/24) from Xunta de Galicia. Part of this work was completed while the first author was visiting the Department of Mathematics, KU Leuven, supported by Xunta de Galicia through the grant ED481A-2019/139. The authors also acknowledge the Supercomputing Center of Galicia (CESGA) for the computational resources.

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Un nuevo contraste de multimodalidad para datos circulares

Diego Bolón [*, **]   Rosa M. Crujeiras [*,**]   Alberto Rodríguez-Casal [*,**]

Las modas de una población son puntos de alta frecuencia que acumulan a su alrededor a la mayor parte de la probabilidad. Obtener información sobre el número de modas de una distribución a partir de una muestra finita puede ser de utilidad incluso cuando los datos no proceden de una variable aleatoria real. Por ejemplo, en [1] emplean un contraste sobre el número de modas de una variable circular para tratar de ver si el patrón estacional natural de los incendios en Galicia está siendo modificado por la actividad humana.

Partiendo de esta base, se propone un nuevo test de multimodalidad para distribuciones circulares. Para ello, se introduce la idea de pseudo-verosimilitud como un análogo no paramétrico de la función de verosimilitud. Esto junto con el concepto de ventana crítica introducido por [2] permite expresar el nuevo contraste con la misma estructura que los test de razón de verosimilitudes paramétricos.

Una vez formalizado el nuevo contraste de multimodalidad, se ha comprobado su comportamiento en la práctica mediante un estudio de simulación. Los resultados muestran como la nueva propuesta ofrece un correcto calibrado, así como una mayor potencia que los principales test de multimodalidad para datos circulares presentes en la literatura.

Como conclusión, se ilustra el funcionamiento del nuevo contraste de multimodalidad aplicándolo a un conjunto de datos reales.

Agradecimientos Este proyecto ha sido financiado por el Ministerio de Universidades a través de las ayudas para la Formación de Profesorado Universitario (FPU20/03960).

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VI Congreso de Jóvenes Investigadores

Real Sociedad Matemática Española

León, Febrero de 2023

Métodos no paramétricos para comparar curvas ROC con covariables

A. Fanjul Hevia[*] W. González Manteiga[**] J.C. Pardo Fernández[***]

La curva ROC (del inglés, Receiver Operating Characteristic curve) es una herramienta estadística que se utiliza para evaluar un sistema de clasificación binario (como puede ser el diagnosis de una enfermedad en sanos/enfermos) [1]. A través de los conceptos de sensibilidad y especificidad, la curva ROC indica en qué medida se consiguen discriminar las dos poblaciones en estudio.

Cuando hay más de un método de clasificación disponible se pueden utilizar sus correspondientes curvas ROC para comparar su comportamiento. Actualmente existe una gran variedad de procedimientos para realizar esa comparación [2]. Sin embargo, la capacidad discriminativa de estos métodos puede verse afectada por la presencia de otras covariables que influyan o bien en las variables que se están empleando para realizar la clasificación o bien en las propias curvas ROC [3]. Dependiendo de la situación en la que se esté se pueden utilizar tres curvas distintas para incorporar el efecto de la covariable al estudio: la curva ROC agregada, la curva ROC condicionada o la curva ROC ajustada.

En este trabajo se emplean métodos no paramétricos para determinar cuál de esas tres curvas debería utilizarse en cada ocasión y, en función del resultado, comparar las curvas correspondientes. Se muestra cómo se estiman dichas curvas, qué metodologías existen para determinar la significación de las covariables [4] y se plantean nuevos contrastes, incluyendo los resultados de un estudio de simulación y su aplicación a un ejemplo con datos reales.

Agradecimientos Los investigadores A. Fanjul Hevia y W. González Manteiga reciben apoyo del Ministerio de Ciencia e Innovación (MCIN/AEI/ 10.13039/501100011033). J.C. Pardo Fernández recibe apoyo del Ministerio de Ciencia e Innovación (MCIN/AEI/10.13039/501100011033). Se agradece al Centro de Supercomputación de Galicia (CESGA) por los recursos computacionales aportados.

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VI Congreso de Jóvenes Investigadores
Real Sociedad Matemática Española
León, Febrero de 2023

The FMM approach for mathematical modelling of the 24-h cardiac rhythm

Y. Larriba [*], C. Rueda [†]

Circadian rhythms are ubiquitous phenomena that recur daily in a self-sufficient, oscillatory manner, along the 24-h day, orchestrating physiological processes such as sleep-wake cycles, feeding, or even cardiovascular function.

The cardiovascular system is an outstanding example of an endogenous circadian pacemaker. Rhythmicity of the heart is observed from genetics to the heart's electrical activity registered by the electrocardiogram (ECG). In particular, long-term metrics measured by ECG, such as RR, PR, or QT segments duration, display 24-h rhythms. These indexes refer to the time elapsed, in milliseconds (*ms*), between the fiducial marks corresponding to the typical ECG waves: P, Q, R, S, and T, and are the key to describing cardiac dynamic in healthy and pathological hearts.

This work is focused on the analysis of heart rate variability (HRV). HRV measures the fluctuation in *ms* between adjacent heartbeats using the RR intervals. HRV generally exhibits a circadian pattern with, at least, a pronounced nocturnal acrophase (a term used to denote the cycle's peak) and a diurnal nadir. Small afternoon acrophases or evening nadirs may also occur. Hence, HRV extraordinarily reflects the non-linearity and complexity of cardiac conduction dynamics being a valuable tool for the prediction of outcomes or detection of changes in clinical status.

Mathematical modelling of HRV rhythms is challenging. Data are usually registered as 24-h ambulatory ECG recording which increases data variability. Data records may still vary depending on the device employed. In addition, HRV patterns are significantly influenced by age, race, sex, physical fitness, clinical conditions, sleep/wake cycles or drug treatment. The attempts in the literature to analyse these data include: polynomial curves, mixed-effect models with several cosine curves, or Fourier decomposition models [1]. Apart from the lack of a universal strategy to analyse HRV data, the methods proposed are usually designed ad-hoc for the problem or database at hand, and model parameters are physiologically meaningless.

To overcome these drawbacks, this work proposes HRV-WA, a genuine FMM-based (Frequency Modulated Möbius) approach, that predicts and assigns the waves involved HRV analysis. FMM approach is a classical signal plus error model, where the signal is decomposed into additive single FMM waves. Each of them are parametrically described in terms of four parameters: A (amplitude), α (wave location) and β (skewness) and ω (kurtosis) [2]. HRV-WA includes: preprocessing, a guided algorithm for HRV analysis, HRV predictions, and the computation of accuracy measures. The performance of the method has been compare on two well-studied datasets for alternative models in the literature.

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León, Febrero de 2023

Testing a linear-circular parametric regression model

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In many scientific fields, such as oceanography, meteorology or biology, data are angular measurements (points in the unit circle of a circular variable), which are accompanied by auxiliary observations of other Euclidean random variables. The joint behavior of these circular and Euclidean variables can be analyzed by considering a linear-circular regression model (circular response and Euclidean covariates), allowing at the same time to explain the possible relation between the variables and also to make predictions on the variable of interest. Parametric regression estimators for linear-circular models have been studied by [1], among others. Alternatively, nonparametric approaches can be used to estimate the unknown regression function. For example, kernel-type regression estimators in a model with a circular response and a $\mathbb{R}^d$-valued covariate have been introduced by [4]. A bandwidth matrix controlling the degree of smoothing has to be selected to compute these kernel-type estimators. If the bandwidth matrix is appropriately chosen, these methods provide more flexible estimators than those obtained using parametric approaches, avoiding misspecification problems. However, if a suitable parametric regression model is assumed, parametric methods usually provide estimators which are more efficient and easier to interpret. In this context, in order to decide if a certain parametric family is appropriate to model the unknown circular regression function, goodness-of-fit tests can be designed.

In this work, new approaches for testing a linear-circular parametric regression model are proposed and analyzed [3]. The test statistic employed in this procedure is based on a comparison between a smoothed parametric fit under the null hypothesis and a nonparametric estimator of the circular regression function. Notice that, in this framework, a suitable measure of circular distance must be employed. The null hypothesis that the regression function belongs to a certain parametric family is rejected if the distance between both fits exceeds a certain threshold. To perform the parametric estimation, procedures based on least squares or maximum likelihood are used [1, 2]. For the nonparametric alternative, a local linear-type estimator [4] is considered. For the application in practice of the proposals, different bootstrap methods are designed and their performance are analyzed and compared in empirical experiments. The testing proposal is also illustrated with a real dataset.

Acknowledgements Research of A. Meilán-Vila and M. Francisco-Fernández has been supported by MINECO (Grant MTM2017-82724-R), MICINN (Grant PID2020-113578RB-I00), and by Xunta de Galicia (Grants ED431C-2020-14 and ED431G 2019/01). Research of R. M. Crujeiras has been supported by MICINN (Grant PID2020-116587GB-I00), and by Xunta de Galicia (Grant ED431C-2021-24).

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León, Febrero de 2023

Probability of default estimation using mixture cure models

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Once the credit scoring assigned by a financial institution to a client who enjoys a personal credit is known, it is interesting to find the probability that the borrower declares himself unable to face the debt contracted with the bank after some time (for example, one year) of its formalization. The main aim of this work is to propose models to estimate this probability, known as the probability of default (PD), using information provided by the credit scoring covariate.

The probability of default conditional on the credit scoring, $PD(t|x)$, can be written as a transformation of the conditional survival function of the variable "time to default", $S(t|x)$:

$$PD(t|x) = 1 - \frac{S(t+b|x)}{S(t|x)}$$

This property is used to propose new PD estimators. Throughout the study of a set of credits, the default is not observed for all of them and the variable "time to default" is censored. As a consequence, censored data and survival analysis are widely used. However, the time to default does not only face a problem of right censoring. Some customers never default, that is, no matter how long you observe such individuals, they will never experience the event of interest. This work discusses techniques for estimating the probability of default (PD) based on cure models, which are the survival models that take into account the existence of a group of cured individuals who are not susceptible to default. A nonparametric survival estimator proposed by [1] and [2] is considered and transformed to obtain an estimator of the probability of default (NPCM estimator). Its behaviour is compared by simulation with Beran's PD estimator based on the generalised limit-product estimator of the conditional survival function and parametric methods based on cure models such as the proportional hazards and the accelerated failure time methods.

The results obtained show that the NPCM estimator provides good estimations of PD and reduces the error committed by the parametric alternatives. Beran's PD estimator is competitive with the NPCM estimator in most scenarios.

The asymptotic properties of the NPCM PD estimator are analysed: an almost sure representation of the estimator and asymptotic expressions of the bias and variance are obtained, as well as its asymptotic normality. Finally, to illustrate the use of Beran's and NPCM estimators, a statistical analysis of German bank loans is carried out.

Acknowledgements This research has been supported by MICINN Grant PID2020-113578RB-100, by the Xunta de Galicia (Grupo de Referencia Competitiva ED431C-2020-14 and Centro Singular de Investigación de Galicia ED431G 2019/01), all of them through the ERDF and by the European Research Council (2016-2022, Horizon 2020 / ERC grant agreement No. 694409). RP was sponsored by inMOTION Programme of grants for pre-doctoral stays Inditex-UDC 2021.

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Survival analysis with a cure fraction and multidimensional covariates

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Standard survival models assume that the event of interest would always happen if there was a sufficient follow-up time. However, this is not always realistic. For instance, HER2-positive breast cancer patients usually receive trastuzumab. Although this therapy has antitumor efficacy, it can cause a problem in the heart, known as cardiotoxicity, in some patients. In this context, there will be a fraction of individuals that will never suffer the side effect, just because they are not susceptible to it. They are said to be cured, in the sense that no matter how long you observe them, they will never experience the final event. To study the time until the cardiotoxicity appears, mixture cure models are appropriate. They allow to estimate both the probability of being cured and the survival function of the uncured population, depending on some covariates. In the literature, nonparametric estimation of both functions is limited to continuous unidimensional covariates [1, 2]. We fill this important gap by considering multidimensional covariates, and proposing a single-index model for dimension reduction. A dataset related to cardiotoxicity from the University Hospital of A Coruña is considered.

Agradecimientos This research has been supported by MINECO grant MTM2017-82724-R, and by the Xunta de Galicia (Grupos de Referencia Competitiva ED431C-2020-14 and Centro de Investigación del Sistema Universitario de Galicia ED431G 2019/01), all of them through the ERDF. The first author acknowledges financial support from Axudas Predoutorais da Xunta de Galicia, with reference ED481A-2020/290, and from Centro de Investigación en Tecnoloxías da Información e das Comunicacións (CITIC) of the University of A Coruña, funded by Xunta de Galicia and the European Union (ERDFGalicia 2014-2020 Program), by grant ED431G 2019/01.

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La correlación de distancias. Aplicación en Análisis de Supervivencia

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En un contexto de análisis de datos en el que existe más de una variable resulta de interés ser capaz de determinar la existencia o ausencia de relaciones de dependencia. Las herramientas clásicas que se utilizan para este fin se encuentran limitadas a la detección de relaciones de dependencia lineal.

En esta charla introduciremos la correlación de distancias, una nueva medida de dependencia propuesta originalmente en [1] aplicable a variables y tipos de dependencia mucho más generales. Veremos como se construye esta magnitud, revisaremos sus principales propiedades y presentaremos dos estimadores empíricos útiles en distintos contextos.

Por último, nos centraremos en una extensión reciente de la propuesta al contexto de Análisis de Supervivencia conforme a lo expuesto en [2]. Veremos su extensión formal y como permite construir un test de independencia basado en permutaciones útil en presencia de censura aleatoria por la derecha. Ilustraremos su desempeño computacional mediante simulaciones.

Agradecimientos María Vidal García es Investigadora Predoctoral en Formación contratada con la ayuda PRE2021-097335, financiado por MCIN/AEI/10.13039/501100011033 y por el FSE+, asociada al proyecto PID2020-116587GB-I00 de la Universidade de Santiago de Compostela (USC).

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Avances y Aplicaciones de la Estadística

Organizadores: Álvaro Méndez, Pablo Morala y Belén Pulido

Lunes 6 de Febrero
Moderadores: *Pablo Morala, Álvaro Méndez Cievieta y Belén Pulido*

12:30 *Integrated Depths for Partially Observed Functional Data*, Antonio Elías.

12:50 *Multivariate epigraph and hypograph indexes. Properties and applications*, Belén Pulido.

13:10 *A quantile based dimension reduction methodology for functional data*, Álvaro Méndez Civieta.

18:15 *Selección de variables en Twin Support Vector Machines*, Sandra Benítez Peña

18:33 *Procedimientos de selección de covariables para modelos de regresión en alta dimensión*, Laura Freijeiro González

18:51 *On the selection of linear and non-linear features in high dimensional additive models*, Manuel Navarro-García

19:09 *Features importance in machine learning: a 2-dimensional approach*, Inmaculada Gutiérrez

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Selección de variables en Twin Support Vector Machines

S. Benítez Peña,* H. Moosaei**

Las máquinas de vectores soporte (SVM) constituyen una herramienta altamente eficaz a la hora de resolver problemas de clasificación binaria ([1]). Por otro lado, las máquinas de vectores soportes gemelas (Twin SVM, [2]) están basadas en la idea de construir SVM cercanos a través de autovalores generalizados (GEPSVM, [3]). Estos, determinan dos hiperplanos no necesariamente paralelos resolviendo dos problemas similares al SVM, pero de menor tamaño, de manera que el coste computacional en la fase de entrenamiento se ve drásticamente reducido en comparación al SVM tradicional. Para ello, se requiere resolver dos problemas de programación cuadrática (QPPs) más pequeños que el QPP que resuelve el SVM estándar. Por otra parte, muchos problemas de la vida real, como aquellos relacionados con la predicción de fraudes y de calificación crediticia, involucran costes de clasificación incorrecta que pueden ser diferentes para ambas clases. A pesar de que proporcionar valores precisos para dichos costes de clasificación incorrecta puede ser dificil para el usuario, no lo es tanto el identificar tasas de clasificación incorrecta aceptables. En este trabajo proponemos un nuevo Twin SVM en el que los costes de clasificación incorrecta son considerados mediante la incorporación de restricciones de rendimiento en la formulación de los problemas. En particular, buscamos los hiperplanos que proporcionen tasas de clasificación incorrecta por debajo de ciertos umbrales. Este modelo es enriquecido realizando a su vez selección de variables (FS), la cual es una tarea fundamental en Ciencia de Datos, haciendo que el método sea más interpretable a la vez que más eficaz. Comenzamos con una breve introducción a la teoría básica de SVM y sobre cómo incorporar FS en este clasificador, usando la técnica presentada en [4]. Tras ello, extendemos esta técnica de FS al clasificador Twin SVM. Se presentan resultados numéricos para conjuntos de datos simulados y reales, mostrando evidencia de la eficacia del método propuesto.

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Integrated Depths for Partially Observed Functional Data

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The classical literature on Functional Data Analysis (FDA) focus on the analysis of curves that are commonly observed in a continuous and compact domain. However, the presence of data sets where the functions are not completely recorded is becoming more recurrent in real applications and, unfortunately, this issue hampers the application of many methodologies for FDA. This phenomenon, termed Partially observed functional data (POFD), is frequently encountered in real applications and is the object of increasing interest in the literature.

We address here the problem of ordering a POFD set from the center to outwards by defining a suitable notion of depth [1]. Particularly, we propose an integrated functional depth for POFD, dealing with the very challenging case where partial observability can occur systematically on any observation of the functional dataset. In particular, differently from many techniques for POFD, we do not request that some functional datum is fully observed, nor do we require that a common domain exist, where all of the functional data are recorded. Because of this, our proposal can also be used in those frequent situations where reconstruction methods and other techniques for POFD are inapplicable. By means of simulation studies, we demonstrate the very good performance of the proposed depth on finite samples, illustrate with examples the intuition of the definition and the important role of the assumptions made about the censoring process.

The notion of depth has demonstrated to be a useful tool for robust and non-parametric statistics; It is not only useful for ranking data sets, but for visualization, outlier detection, and classification [1], for forecasting [2] and for missing values imputation [3], among others. Our proposal enables the use of benchmark methods based on depths, originally introduced for fully FDA, in the case of POFD.

As a case study, we consider the AneuRisk dataset that has been analysed for evaluating the role of vascular geometry and hemodynamics in the pathogenesis of cerebral aneurysms. Our goal is to discriminate patients with aneurysms in different districts by using the geometric features of the internal carotid artery expressed by its radius profile and centreline curvature. These two variables are partially observed, and many analyses had to be restricted to the domain where all the individuals were recorded. We show that using our definition of the partially observed sample and DD-classifiers, the classification errors obtained are smaller than in other studies.

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Procedimientos de selección de covariables para modelos de regresión en alta dimensión

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En un modelo de regresión se intenta explicar una variable de interés, $Y \in \mathbb{R}$, a través de un conjunto de variables explicativas $X = (X_1, \ldots, X_p)^\top \in \mathbb{R}^p$. Sin pérdida de generalidad, asumiendo que tanto Y como X están centradas, dicha relación se puede expresar mediante

$$Y = m(X) + \varepsilon \tag{1}$$

donde ε es el error no observable del modelo y $m(\cdot)$ es una función desconocida a priori que, en general, recoge la información de la esperanza condicional, i.e. $m(X) = \mathbb{E}\left[Y|_X\right]$.

A la hora de ajustar el modelo, estimando la función $m(\cdot)$ de (1), cuando el número de covariables, p, es grande o incluso mayor que el número de muestras observadas, n, aparecen problemas. Muchos métodos usuales dejan de funcionar adecuadamente en este contexto y nuevas técnicas son requeridas. Así mismo, la inclusión de muchas covariables en el modelo puede producir los conocidos como efectos espúreos, considerando como importantes covariables que en realidad no lo son. Con el objetivo de reducir el número de variables explicativas y garantizar que las seleccionadas son relevantes, consiguiendo un modelo más harmonioso, es necesario recurrir a técnicas de selección de covariables aptas para este contexto.

A lo largo de los últimos treinta años, se han propuesto en la literatura diversas técnicas para la selección de covariables en este contexto de alta dimensión con $p > n$. En este trabajo haremos una revisión de las mismas, distinguiendo los casos donde la estructura del modelo en (1) es establecida de antemano, como es el caso del modelo lineal con penalización L_1 (la llamada regresión LASSO [1]), o cuando la forma del modelo no es conocida para lo que extenderemos las ideas de la correlación de distancias ([2]).

Agradecimientos Esta investigación está financiada por la Consellería de Cultura, Educación e Ordenación Universitaria junto con la Consellería de Economía, Emprego e Industria de la Xunta de Galicia (project ED481A-2018/264). Así mismo, por el Proyecto PID2020-116587GB-I00 fundado por MCIN/AEI/10.13039/501100011033, por "ERDF A way of making Europe", junto con los Grupos Competitivos de Referencia 2021-2024 (ED431C 2021/24) de la the Xunta de Galicia mediante el ERDF.

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Features importance in machine learning: a 2-dimensional approach

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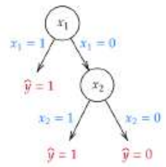
An important issue in machine learning analysis is about the prediction accuracy. Besides this presupposed ability of a "good" model, interpretability should not be forgotten. Many authors address the problem of feature importance measurement explaining machine learning models with game theory tools. The Shapley value [1] is used to evaluate the prediction of each feature, to assign them an importance [2]. The common point of previous proposals is that the solution is a crisp value. Our contribution goes beyond with the definition of 2-dimensional solutions. We illustrate it. Table 1a is a sample of any database with a dependent feature y, and two independent variables, x_1 and x_2; in which $25\,\%$ of the instances are like the first row, $25\,\%$ of the instances are like the second row, and so on. Specifically, the model considered is the decision tree below. Due to the relations among features, classic methods provide aggregated predictions, so we cannot distinguish the case in which we do not know all the features. With [2] as starting point, we propose a disaggregated model which allows to analyze the importance of the features with all the information (known features). If x_1 is known but x_2 is not, classic models cannot decide in the second branch of the tree. The same happens in the first branch if x_1 is unknown and x_2 is known; and in case both are unknown. x_1 does not contribute the same if x_2 is known or not, and vice versa. We can affirm the importance features depends on the whole available information. With our model, it is easy to consider, quantify and represent this situation. We work with the Shapley value [1] and propose a solution in terms of game theory, and also in terms of fuzzy measures [3], about which the Shapley value as also been characterized [4]. As this value can be seen as an aggregation, we work in the decomposition of its components, focusing on the sum. Other aggregations will be considered to obtain different importance measures, used to quantify the importance of features.

(a) Database.

row	probability	y	x_1	x_2
First	$0{,}25\,\%$	1	1	0
Second	$0{,}25\,\%$	1	1	1
Third	$0{,}25\,\%$	1	0	1
Fourth	$0{,}25\,\%$	0	0	0



(b) Decision tree.

Acknowledgements This research has been partially supported by the Government of Spain, Grant Plan Nacional de I+D+i, PID2020-116884GB-I00, PID2021-122905NB-C21.

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A quantile based dimension reduction methodology for functional data

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The majority of methodologies in Functional Data Analysis (FDA) treat the curves as smooth functions observed with error, and are centered in modelling the mean curves. However, this approach does not take into account the within subject variability and correlation of the data, as those cannot be reflected by a expected value. The work presented here addresses this gap and seeks to capture the within subject variability by modelling the subject specific conditional quantiles. This objective lays on the assumption of treating each subject as a single realization from its own underlying distribution. When this distribution is symmetric and there are no outliers in the data, the expected value provides very good results. However, in many applications the distribution is skewed, with changes over the functional domain that are not reflected in the expected value but can be reflected modelling the conditional quantiles.

We introduce here the functional quantile principal component analysis (FQPCA), a dimensionality reduction technique that extends the concept of Functional Principal Components (FPCA) [2] to the quantile regression framework [1], obtaining a model that can explain the subject specific quantiles conditional on a set of loading functions. FQPCA is able to capture shifts on the scale and distribution of the data that may affect the quantiles but may not affect the mean, and is also a robust methodology suitable for dealing with outliers, heteroscedastic data or skewed data. The model is estimated using penalized splines, and can deal with sparse and irregular time measurements, usually known as Partially Observed data.

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Evaluation and challenges of interpretability methods for feature interactions in neural networks

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During the past years, neural networks have become one of the most used and developed machine learning tools, specially with the constant advances in deep learning. However, their lack of interpretability and black box nature remains a significant challenge and explainable artificial intelligence is a topic of active research. In light of these challenges, there has been growing interest in finding ways to combine neural networks with more traditional statistical methods to create alternative representations that can help address these issues.

In this context, NN2Poly [1] [2] is a method that, under certain constraints, can obtain an alternative representation as a polynomial of a given fully connected neural network (or multilayered perceptron). To do so, Taylor expansion is employed to approximate the activation function at each neuron and layer. Then, using the weights and some combinatorial properties related with multisets partitions, polynomials in the original features are obtained at each neuron and layer up to the final output. This alternative representation can help interpret the neural network output, as each polynomial coefficient is associated with a combination (or interaction) of features or variables, and the total number of parameters is significantly reduced. In this sense, it acts as a global explainability method for the neural network. Additionally, using this classical statistical model to represent neural networks can be useful to study their properties employing a different framework with a distinct and novel perspective.

In this talk we will see how to use NN2Poly to explore how interactions of features that are learnt by a neural network and the main similarities and differences that arise when comparing the obtained explanations with well established explainability methods like LRP (Layerwise Relevance Propagation) [3] or SHAP (SHapley Additive exPlanations) [4]. These two methods tackle the problem of explainability in a different manner than NN2Poly, as both are local methods (explanation for each prediction) instead of global (explanation for the whole model at once), and they assign the importance to individual features, thus not explicitly considering interactions. However, we explore ways to use them to also understand what the neural network has learnt as a global model. Simulations with polynomial data where the interactions are known will be used to study these ideas and analyse the challenges that appear when extending them to more complex cases.

Acknowledgements This research is part of the I+D+i projects PID2019-104901RB-I00 and PID2019-106811GB-C32, funded by MCIN/AEI/10.13039/501100011033/.

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VI Congreso de Jóvenes Investigadores

Real Sociedad Matemática Española

León, Febrero de 2023

On the selection of linear and non-linear features in high dimensional additive models

Manuel Navarro-García[*] Vanesa Guerrero[†] María Durbán[‡] Arturo del Cerro[§]

Sparse feature selection plays an important role in modern regression analysis, which aims to produce interpretable models without compromising its performance. Central to this statistical task lies the best subset selection problem with subset size k [4], whose aim is to identify the k covariates which produce the best fit in terms of an empirical loss function. The cardinality constraint makes this problem non-convex, which has led many researchers to propose alternative approaches to avoid the computational difficulties that such a decision problem entails. In this context, most of the existing techniques either assume linearity on the predictors and/or use convex penalties to gain sparsity in the prediction model. However, the linearity assumption might be a poor approximation of reality, and considering the cardinality constraint in a soft manner may lead to select a large number of non-informative features.

In this work, we address the problem of finding the best subset of variables from a given set in a non-parametric regression setting. To capture the underlying non-linear relationships between each feature and the response variable, additive models are used, in which the smooth terms are approximated by means of $B-$splines bases [2]. These expansions generate a group structure on the parameters to be estimated, so the initial best subset problem becomes a best group subset problem. To provide more flexibility to the model and make it more interpretable, each smooth term is decomposed into a linear and a non-linear term, and hence allowing the variables to enter in the model as linear and/or non-linear. The full model is fitted using the $P-$splines approach [3] and subject to the cardinality constraint, yielding a mixed integer quadratic optimization problem. We compare our approach with other state-of-the-art algorithms (such as [1] and [5]) on both synthetic and real-world data sets.

Acknowledgements The research of Manuel Navarro-García is supported by the research projects IND2020/TIC-17526 (Comunidad de Madrid) and PID2019-104901RB-I00 (funded by MCIN/AEI/10.13039/501100011033). This support is gratefully acknowledged.

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VI Congreso de Jóvenes Investigadores

Real Sociedad Matemática Española

León, Febrero de 2023

Multivariate epigraph and hypograph indexes. Properties and applications

A. Belén Pulido Bravo [*] B. Alba M. Franco-Pereira [*†] C. Rosa E. Lillo [*‡]

An important question when dealing with functional data is how to order functions. There are different possibilities for overcoming this issue. One could think on the concept of statistical depth, which has been defined in few different ways and gives an ordering from center to outwards. Nevertheless, if one is looking for something more similar to the natural order in $\mathbb{R}$, the epigraph and the hypograph indexes could be considered to order functions from top to bottom or bottom to top. These indexes are fully explained in [1], where some properties are given.

If instead of studying one single variable, one needs to deal with the relation between two or more variables, the concept of multivariate functional data arises. For example, studying the growth of a children population would be considered as functional data analysis in one dimension, while studying both height and weight of this population at the same time would be included into multivariate functional data analysis.

In [2], the concept of band depth for functional data is extended for multivariate functional data and some properties of these indexes are explained. A multivariate version of the epigraph and the hypograph is proposed in [3]. This new definition is based on calculating the epigraph or the hypograph for each component, and the multivariate indexes are given as an average of them. Here, we propose an alternative definition for the multivariate epigraph and hypograph indexes, which takes into account the multidimensionality of the data. In addition, some important properties that these indices fulfil will be discussed. Moreover, the differences between ordering component by component and considering all of them at the same time will be presented. Finally, some application of these indexes will be introduced.

Acknowledgements This research has been partially supported by Ministerio de Ciencia e Innovación, Gobierno de España, grant numbers PID2019-104901RB-I00, PID2019-104681RB-I00 and PTA2020-018802-I.

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Teoría de la Probabilidad

Organizador: Ignacio Montes

Martes 7 de Febrero
Moderador: *Ignacio Montes*

10:00 *Procesos de ramificación defectuosos en ambiente variable*, Carmen Minuesa

10:30 *Un paseo aleatorio entre modelos epidemiológicos*, Giulia Binotto

11:30 *On approximate validation of models: a Kolmogorov–Smirnov-based approach*, Hristo Inouzhe

12:00 *Coeficientes de asimetría estadística asintóticamente normales*, Raúl Pérez-Fernández

12:30 *Making data fair through optimal transport*, Paula Gordaliza

13:00 *Un análisis de robustez Bayesiana multivariante con enfoque en principios de primas*, Marta Sánchez-Sánchez

18:15 *Conjuntos credales representables mediante intervalos de probabilidad alcanzables y funciones de creencia*, Serafín Moral-García

18:45 *Determinando si un conjunto aleatorio es conexo*, Juan-Jesús Salamanca

19:15 *EM for Approximating Unidentifiable Counterfactual Queries*, Rafael Cabañas

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Real Sociedad Matemática Española

León, Febrero de 2023

Un paseo aleatorio entre modelos epidemiológicos

G. Binotto*

Los modelos matemáticos aplicados al campo de la biología y, en particular, de la epidemiología tienen actualmente un gran interés a causa del brote generado por el COVID-19. No obstante, su estudio para describir la propagación de una enfermedad tiene una larga historia. Se remonta a la investigación sobre la inoculación de la viruela llevada a cabo por Bernoulli en el siglo XVIII [1]. Desde entonces se han considerado tanto modelos deterministas como estocásticos y se han tenido en cuenta muchos factores: agentes infecciosos, modo de transmisión, periodos de latencia, inmunidad temporal o parcial, periodos de cuarentena, etc.

El objetivo de esta ponencia es presentar algunos de los modelos epidemiológicos más importantes que se han presentado al largo de la historia, hasta llegar a un modelo estocástico más reciente, propuesto por Tuckwell y Williams en 2007 [2]. Este hace parte de los modelos de tipo SIR, que dividen la población en tres diferentes clases: Susceptibles, Infectados y Recuperados (*Removed*, en inglés), respectivamente. Es un modelo a tiempo discreto en el cual la población total permanece constante y los individuos se encuentran con un número aleatorio de otros individuos en cada paso de tiempo.

Para terminar, se verán extensiones de este modelo en los cuales se añaden nuevos factores y se tiene en cuenta la dependencia del tiempo de algunos parámetros. Con la ayuda de algunas simulaciones, se mostrará como la evolución de una enfermedad se ve afectada por esta dependencia temporal. Estos últimos resultados forman parte de un trabajo realizado con M. Besalú [3].

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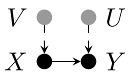
VI Congreso de Jóvenes Investigadores
Real Sociedad Matemática Española
León, Febrero de 2023

EM for Approximating Unidentifiable Counterfactual Queries

Rafael Cabañas * Marco Zaffalon † Alessandro Antonucci †

Causality is an emerging direction for data science with applications in diverse domains which allows to reason about hypothetical scenarios. Structural causal models (SCMs)[1] are a class of probabilistic graphical model for causality made of endogenous and exogenous variables. Frequently, the exogenous probabilities are not available (due to the lack of data for these variables). Thus, many causal and counterfactual queries are not identifiable. For solving this, a previous work [2] proposes an exact mapping between SCMs and credal networks by specifying a set of linear constraints for the exogenous probabilities. However this method is not always applicable. Here we summarize a recent work [3] proposing an alternative procedure for specifying the exogenous credal sets based on the *expectation-maximization (EM)* algorithm.

In a SCM, the causal relations are expressed in the form of a directed acyclic graph (DAG) in which the exogenous ones are the root nodes. Figure 1 shows an example of a SCM with two endogenous variables X and Y. The relation between the exogenous and endogenous variables is given by the structural equations (i.e., deterministic functions). We assume that data from the endogenous variables is available and hence the empirical distributions $\tilde{P}(Y|X)$ and $\tilde{P}(X)$ can be estimated.



$$f_X(V) = \begin{array}{cc} \begin{array}{cc} v_1 & v_2 \end{array} & \\ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} & \begin{array}{c} x_1 \\ x_2 \end{array} \end{array} \quad f_Y(U,X) = \begin{array}{cc} \begin{array}{cccc} u_1 & u_2 & u_3 & u_4 \end{array} & \\ \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix} & \begin{array}{c} y_1, x_1 \\ y_2, x_1 \\ y_1, x_2 \\ y_2, x_2 \end{array} \end{array}$$

Figure 1: Example of a simple SCM.

The exact transformation of a SCM requires to specify a set of linear constraints for each exogenous variable. In the previous example, the credal set $K(V)$ is defined by the constraints given in Figure 2 (left). Let's assume that in the limit of infinite data we have that $\tilde{P}(y_1|X) = [0.4, 0.6]$, then the feasible solution is the line depicted in Figure 2 (right). When dealing with finite sampled data we might obtain $\tilde{P}(y_1|X) = [0.4, 0.58]$, for which the constraints cannot be solved. In this case, we can proceed as follows: $P(U)$ is randomly initialized and EM is executed . If the process is repeated multiple times, $K(V)$ is defined by all the values for $P(V)$ obtained at convergence. The ten red points in Figure 2 (right) are the result of applying EM with the incompatible data. Once the specification for the credal set is available, any causal query can be calculated if the proper graphical transformations are done. In case of the EM-based solution, the causal query is run independently for each precise model and then combined. In the example, the bounds for the counterfactual query $P(Y_{x_1} = y_1|x_1)$ are $[0.41, 0.59]$ while with the exact approach these are $[0.4, 0.6]$.

$$\begin{aligned} P(u_1) + P(u_2) + P(u_3) &= \tilde{P}(y_1|x_1)\,, \\ P(u_3) &= \tilde{P}(y_2|x_1)\,, \\ P(u_3) + P(u_4) &= \tilde{P}(y_1|x_2)\,, \\ P(u_1) + P(u_2) &= \tilde{P}(y_2|x_2) \end{aligned}$$

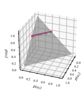


Figure 2: Specification of the credal set $K(V)$.

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VI Congreso de Jóvenes Investigadores
Real Sociedad Matemática Española
León, Febrero de 2023

Making data fair through optimal transport

Paula Gordaliza[*†] Eustasio del Barrio[†] Fabrice Gamboaz[‡] Hristo Inouzhe[*]
Jean-Michel Loubes[‡]

The aim of this talk is two-fold. The first part is devoted to the asymptotic theory of the empirical transportation cost where optimal transportation methods are studied for statistical inference purposes. We provide a Central Limit Theorem for the Monge-Kantorovich distance between two empirical distributions with different sizes n and m, $\mathcal{W}_p(P_n, Q_m)$, $p \geq 1$, for observations on $\mathbb{R}$. In the case $p > 1$ our assumptions are sharp in terms of moments and smoothness. We prove results dealing with the choice of centering constants. We provide a consistent estimate of the asymptotic variance which enables to build two sample tests and confidence intervals to certify the similarity between two distributions. Additionally, a moderate deviation principle for the empirical transportation cost is obtained in general dimension.

In the second part, we present an application of this theoretical results to the recent fair learning problem. Algorithmic fairness is one of the main concerns of today's scientific society due to the generalization of predictive algorithms in all aspects of human life. We motivate the fairness problem by presenting some comprehensive results from the study of the *demographic parity* (DP) criterion through the analysis of the *disparate impact* index on the real and well-known *Adult Income* dataset. Importantly, we show that trying to make fair machine learning models may be a particularly challenging task, especially when the training observations contain bias. Then a review of mathematical models for fair learning is given in a general setting, with some novel contributions in the analysis of the price for fairness in regression and classification. Moreover, we recast the links between fairness and predictability in terms of probability metrics. After this review part, we present our contributions to fair learning using optimal transport.

The goal is to check if there is group bias in the response variable Y with respect to a sensitive information S present in the data. However, not all individuals in S are comparable, and some differences in the target Y may arise from genuine differences in the data. We study two different kind of methods: repairing and trimming. In the first case, we analyze data repair methods based on mapping conditional distributions to the Wasserstein barycenter that ensure total fairness in terms of DP. Additionally, we propose a *random repair* which yields a tradeoff between minimal information loss and fairness. Finally, in the classification setting, we build a tool for global fairness assessment of a dataset based on the two-sample similarity tests mentioned above. In contrast, the second approach aims at making data fair though optimal trimmed matching. Precisely, we propose to eliminate such cases by trimming an α proportion of the input data as a pre-processing step to any further learning mechanism in order to obtain the two closest possible marginal distributions (w.r.t. S). On this population that is 'similar enough' we can check for discrimination, in the sense of DP. We solve a trimmed matching problem subject to fairness constraints that is a linear program that can be addressed with well-known techniques. We present some successful results of application to synthetic and real data.

Agradecimientos This research was funded by the Basque Government through the BERC 2022-2025 program and Elkartek project 3KIA (KK-2020/00049), and by the Spanish Ministry of Science, Innovation, and Universities (BCAM Severo Ochoa accreditation SEV-2017-0718).

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León, Febrero de 2023

On approximate validation of models: a Kolmogorov–Smirnov-based approach

E. del Barrio [*], H. Inouzhe [†], C. Matrán [**] [‡]

Classical tests of fit typically reject a model for large enough real data samples. In contrast, often in statistical practice, a model offers a good description of the data even though it is not the 'true' random generator. We consider a more flexible approach based on α-contamination neighbourhoods which were introduced by Huber in [1].

An α-contamination neighbourhood of a probability distribution P_0 is the set of probability distributions $\mathrm{V}_\alpha(P_0) = \{(1-\alpha)P_0 + \alpha Q : Q \in \mathcal{P}\}$, where $\mathcal{P}$ is the set of all probability distributions in the space (throughout this work probabilities on the real line $\mathbb{R}$). Contamination neighbourhoods are nicely related to trimmings by (see [2]) P$\in \mathcal{V}_\alpha(P_0) \iff P_0 \in R_\alpha(P)$, where $R_\alpha(P)$ denotes the set of α-trimmings of the probability distribution P, $\mathrm{R}_\alpha(P) = \left\{Q \in \mathcal{P} : Q \ll P,\ \frac{dQ}{dP} \leq \frac{1}{1-\alpha}\ P\text{–}a.s.\right\}$.

Using trimming methods and the well-known Kolmogorov metric between two cumulative distribution functions F and G

$$d_K(F,G) = \sup_{x\in\mathbb{R}} |F(x) - G(x)|,$$

we introduce a functional statistic measuring departures from a contaminated model. We show how the plug-in estimator $d_K(F_0, R_\alpha(F_n))$ allows testing of fit for the (slightly) contaminated model (F_0) vs sensible deviations from it, i.e., $\mathrm{H}_{0,n} : d_K(F_0, R_\alpha(F)) = 0 \quad vs. \quad H_{1,n} : d_K(F_0, R_\alpha(F)) > \rho_n,$ for $\rho_n = \rho(n) > 0$, with uniformly exponentially small type I and type II error probabilities.

We also address the asymptotic behaviour of the estimator showing that, under suitable regularity conditions,

$$\sqrt{n}\,(d_K(F_0, R_\alpha(F_n)) - d_K(F_0, R_\alpha(F)))$$

asymptotically behaves as the supremum of a Gaussian process.

As an application, we explore methods of comparison between descriptive models based on the paradigm of model falseness. We also include some connections of our approach with the false discovery rate setting, showing competitive behaviour when estimating the contamination level, and being applicable in a wider framework. The interested reader can find further details in [3] and [4].

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VI Congreso de Jóvenes Investigadores

Real Sociedad Matemática Española

León, Febrero de 2023

Procesos de ramificación *defectuosos* en ambiente variable

C. Minuesa,* G. Kersting**

En este trabajo nos centramos en la modelización de poblaciones hasta la aparición por primera vez de una característica mostrada por los individuos, como puede ser una mutación o enfermedad, supuesto que la población comienza con individuos que no la presentan. En concreto, consideramos el caso en el que en cada generación todos los individuos se reproducen de manera independiente al resto y de acuerdo a la misma distribución de probabilidad, conocida como ley de reproducción, la cual puede variar a lo largo de las generaciones.

Esta situación puede describirse a través de procesos estocásticos conocidos como *procesos de ramificación* defectuosos *en ambiente variable* (PRDAV, o *defective Galton-Watson processes in a varying environment* en inglés) introducidos en [1]. Un PRDAV es una cadena de Markov que queda determinada por las leyes de reproducción en cada generación o equivalentemente, a través de $v = \{f_1, f_2, \ldots\}$, donde f_n denota la función generatriz de probabilidad de la ley de reproducción, posiblemente impropia, en la generación n-ésima. De esta forma, $1 - f_n(1)$, puede interpretarse como la probabilidad de que en la generación n, un individuo envíe el proceso a un estado absorbente Δ en la generación $n+1$, donde el proceso permanece para siempre. Como consecuencia, su espacio de estados es $\mathbb{N}_\Delta = \mathbb{N}_0 \cup \{\Delta\}$, donde dos de estos estados son absorbentes: 0 and Δ. En este trabajo estudiamos el comportamiento asintótico de un PRDAV. Se presentarán los dos resultados centrales relativos al comportamiento límite de un PRDAV: la convergencia casi segura del proceso a una variable aleatoria con valores en $\mathbb{N}_\Delta \cup \{\infty\}$ y dos caracterizaciones de la dualidad extinción-absorción en Δ. Estos extienden los resultados ya presentados en [2] para el caso en que la ley de reproducción es constante a lo largo de las generaciones.

Agradecimientos Esta línea de investigación forma parte del proyecto PID2019-108211GB-I00, concedido por la Agencia Estatal de Investigación MCIN/AEI/10.13039/501100011033/.

Este tema de investigación fue iniciado cuando Carmen Minuesa visitó el Instituto de Matemáticas, de la Universidad Goethe Fráncfort del Meno, y está agradecida por la hospitalidad y colaboración

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VI Congreso de Jóvenes Investigadores
Real Sociedad Matemática Española
León, Febrero de 2023

Conjuntos credales representables mediante intervalos de probabilidad alcanzables y funciones de creencia

S. Moral-García*

La teoría clásica de la Probabilidad (TP) es la forma estándar de representar la información probabilística sobre un conjunto finito de alternativas proporcionada por un experto o conjunto de datos. Sin embargo, en muchos casos, una única distribución de probabilidad no es adecuada porque no hay información suficiente debido a errores o incompletitud en los datos.

Por este motivo, se han desarrollado en la literatura teorías basadas en probabilidades imprecisas. Un resumen de ellas puede verse en [1]. Todas ellas generalizan la TP. Hay teorías de probabilidades imprecisas que son más generales que otras, y también hay pares de teorías de probabilidades imprecisas tales que ninguna de ellas generaliza la otra. Dado que estas teorías tienen propiedades matemáticas específicas, algunas de ellas son más adecuadas que otras en situaciones concretas.

Por un lado, la Teoría de la Evidencia (TE) [2, 3] se ha usado frecuentemente para tratar información basada en incertidumbre en aplicaciones prácticas. En la TE, el conocimiento probabilístico sobre el conjunto de alternativas viene determinado por una función de creencia. Por otro lado, los intervalos de probabilidad alcanzables [4], que proporcionan una cota de probabilidad inferior y superior sobre la probabilidad de cada alternativa, son fáciles de entender y manejar y tienen gran poder expresivo. Se han usado con éxito en aplicaciones prácticas como clasificación.

Como se demuestra en [5], la TE y los intervalos de probabilidad alcanzables son paralelos en cuanto a generalidades, ya que un conjunto de intervalos de probabilidad alcanzables no siempre se puede representar mediante funciones de creencia ni una función de creencia tiene asociada siempre un conjunto de intervalos de probabilidad alcanzables.

En este trabajo, estudiamos la relación entre la TE y los intervalos de probabilidad alcanzables. Para esto, proporcionamos un conjunto de condiciones necesarias y suficientes bajo las cuales un conjunto de intervalos de probabilidad alcanzables es representable mediante una función de creencia, así como una condición necesaria y suficiente para que una función de creencia tenga asociado un conjunto de intervalos de probabilidad alcanzables.

Agradecimientos Este trabajo ha sido financiado por los fondos UGR-FEDER con el proyecto A-TIC-344-UGR20, y por la JUnta “Junta de Andalucía-Consejería de Transformación Económica, Industria, Conocimiento y Universidadesçon el proyecto Project P20_00159.

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VI Congreso de Jóvenes Investigadores

Real Sociedad Matemática Española

León, Febrero de 2023

Coeficientes de asimetría estadística asintóticamente normales

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La *simetría* es una característica de un objeto que permanece invariante tras ser expuesto a una transformación geométrica. La presencia de esta característica ha sido estudiada desde tiempos inmemoriales en contextos a priori tan distintos como pueden ser el arte, la naturaleza y las matemáticas. En el campo de la probabilidad y la estadística, la simetría de una variable aleatoria X implica que la distribución de $X - x_0$ y de $x_0 - X$ es la misma para un cierto $x_0 \in \mathbb{R}$. Un ejemplo común en el que esta propiedad resulta ser de gran utilidad es en el uso de tablas estadísticas de distribuciones simétricas, como es por ejemplo el caso de la distribución normal, donde podemos calcular fácilmente probabilidades acumuladas en la cola derecha a partir de las probabilidades acumuladas en la cola izquierda.

Los primeros esfuerzos en medir la asimetría (i.e., la falta de simetría) de una variable aleatoria se suelen atribuir a Pearson en el siglo XIX, donde se presentan los primeros ejemplos de lo que hoy en día son conocidos como coeficientes de asimetría. Sin embargo, la formalización axiomática del concepto de coeficiente de asimetría es fruto de numerosos estudios posteriores donde destacan los trabajos de van Zwet [5] y Oja [4]. De manera formal, se entiende como coeficiente de asimetría a una función $\gamma : \mathcal{F} \to \mathbb{R}$ que asigna a cada variable aleatoria X un grado de asimetría cumpliendo, al menos, las tres propiedades siguientes: (i) $\gamma(X) = 0$ siempre que X sea una variable aleatoria simétrica; (ii) $\gamma(a\,X + b) = \gamma(X)$, para todo $a, b \in \mathbb{R}$ con $a > 0$; y (iii) $\gamma(-X) = -\gamma(X)$. Otros axiomas adicionales han sido explorados por diversos autores, sin embargo no suelen englobar a todas las funciones utilizadas en la literatura con el fin de medir el grado de asimetría de una variable.

Una gran cantidad de coeficientes de asimetría han sido estudiados en la literatura, desde ejemplos más clásicos como los coeficientes de asimetría de Fisher, Pearson o Bowley, hasta ejemplos más recientes como es el caso de la medcouple [1]. Todos estos coeficientes de asimetría tienen asociada una versión muestral que, bajo condiciones más o menos restrictivas, tiene una distribución asintótica normal, centrada en el valor poblacional. Esta normalidad asintótica, unida al axioma (ii) de la definición de coeficiente de asimetría, nos permite utilizar un coeficiente de asimetría para definir un test de bondad de ajuste a una familia de localización-escala (véase [1]). Desafortunadamente, puesto que la varianza asintótica depende de la distribución subyacente, definir un test de simetría no es tarea sencilla. Diferentes autores han tratado de evitar este problema considerando una distribución de referencia o simplemente estimando la varianza asintótica (véase [2]). En esta última dirección, en el presente trabajo se persigue el uso del bootstrap [3, 6] para estimar dicha varianza asintótica en combinación con diferentes coeficientes de asimetría. Este proceso resulta en la definición de diversos tests de simetría, la mayoría de ellos no estudiados en la literatura con anterioridad, para los que se realiza un estudio comparativo de la potencia en distintas distribuciones (tanto simétricas como asimétricas).

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VI Congreso de Jóvenes Investigadores

Real Sociedad Matemática Española

León, Febrero de 2023

Determinando si un conjunto aleatorio es conexo

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Los conjuntos aleatorios son modelos probabilísticos que han aparecido en diversos contextos (véase, por ejemplo, [3] para profundizar sobre el papel de estos modelos en diversas situaciones estadísticas).

Fíjese un espacio topológico $\mathcal{M}$ con ciertas propiedades topológicas. Un *conjunto aleatorio* (*cerrado*) $\widehat{X}$ sobre $\mathcal{M}$ es una aplicación desde un espacio de probabilidad completo hasta la clase de conjuntos cerrados de $\mathcal{M}$ y que ha de satisfacer cierta hipótesis de medibilidad, [2]. La definición de conjunto aleatorio replica la idea de elemento aleatorio como la de variable aleatoria o vector aleatorio, estando aquí el espacio muestral representado por la clase de cerrados de $\mathcal{M}$. El axioma de medibilidad es el necesario para que estén bien definidas las probabilidades de que $\widehat{X}$ interseque a cualquier conjunto compacto K de $\mathcal{M}$, [2]. Matemáticamente, $P(\widehat{X} \cap K \neq \emptyset)$ siempre estará bien definido para todo compacto K. De este modo podemos definir la *capacidad* $T_{\widehat{X}}$ de $\widehat{X}$ como la aplicación que asocia a cada compacto K de $\mathcal{M}$ la probabilidad anterior. El famoso teorema de Choquet-Kendall-Matheron establece que todo conjunto aleatorio queda caracterizado por su capacidad. Es decir, que la capacidad de un conjunto aleatorio es a un conjunto aleatorio lo que la función de distribución es a una variable aleatoria. Equivalentemente, que toda la información probabilística de un conjunto aleatorio la preserva su capacidad. En consecuencia, en teoría se ha de caracterizar toda propiedad topológica de un conjunto aleatorio a través de la capacidad de dicho conjunto aleatorio. En particular, si es conexo.

Nuestro objetivo es determinar si un conjunto aleatorio dado es conexo atendiendo únicamente a su capacidad. Observar que la capacidad es una aplicación sobredimensionada, en el sentido de que su espacio inicial lo conforma la clase de compactos de $\mathcal{M}$. Esto supone un escollo grave en principio, pues idealmente deseamos poder resolver el problema planteado sobre conexión de una manera algorítmica y en tiempo finito.

El resultado principal que expondremos caracteriza la conexión de un conjunto aleatorio mediante su capacidad. De hecho, se demuestra que, bajo ciertas hipótesis razonables, un conjunto aleatorio es conexo si y sólo si su capacidad satisface cierta ecuación lineal cuyos términos son las capacidades evaluadas en cierto número finito de conjuntos, [4, 5].

La presentación pretenderá exponer algunas ideas sobre la demostración de los resultados matemáticos principales.

Agradecimientos Proyecto PGC2018-098623-B-I00.

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VI Congreso de Jóvenes Investigadores

Real Sociedad Matemática Española

León, Febrero de 2023

Un análisis de robustez Bayesiana multivariante con enfoque en principios de primas

F. Ruggeri[*] M. Sánchez-Sánchez[**] A. Suárez-Llorens

En esta comunicación utilizaremos una nueva metodología que nos permitirá estudiar la sensibilidad Bayesiana incorporando incertidumbre a través de las clases de distribuciones a priori que encontramos en [1], donde usan densidades ponderadas para inducir incertidumbre sobre la información a priori. En un enfoque actuarial y debido a la particular forma de dichas clases, podremos obtener cotas superiores e inferiores tanto para las primas colectivas o primas priori como para las primas Bayesianas o primas a posteriori. Esto será posible gracias a las propiedades de preservación del orden de las distribuciones a priori y posteriori a través de ordenaciones estocásticas multivariantes que tienen algunos principios de primas. Adicionalmente, veremos como esta metodología tiene aplicaciones en los sistemas de tarificación Bonus-Malus. Para finalizar, podremos ver la aplicación de estos resultados en un conjunto de datos actuariales.

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Análisis Funcional

Organizadores: Luis Carlos García y Gonzálo Martínez

Miércoles 8 de Febrero
Moderador: *Christian Cobollo*

11:30 *On the numerical index of the real two-dimensional L_p space*, Alicia Quero de la Rosa

12:00 *Banach spaces contained in the set of strongly norm-attaining Lipschitz mappings*, Óscar Roldán Blay

13:00 *La convolución y la propiedad de la diferenciabilidad*, Pablo Jiménez Rodríguez

Jueves 9 de Febrero
Moderadores: *Alberto Salguero Alarcón y Alicia Quero de la Rosa*

10:00 *Sumas torcidas de espacios $c_0(I)$*, Alberto Salguero Alarcón

10:30 *Derivación de funtores en espacios de Banach*, Nazaret Trejo Arroyo

11:30 *El problema del subespacio complementado en retículos de Banach*, David de Hevia Rodríguez

12:00 *Dualidad en retículos de Banach libres*, Enrique Sánchez Pérez

12:30 *Funciones uniformemente convexas y teorema de Enflo*, Guillaume Grelier

13:00 *Geometría local vs geometría global en la esfera unidad de un espacio de Banach*, Andrés Quilis

17:00 *Convergence and spaceability within the disc algebra*, Pablo José Gerlach Mena

17:30 *Holomorphic functions attaining their weighted norm*, Rubén Medina Sabino

18:00 *Phelps Property U and $C(K)$ spaces*, Christian Cobollo Gómez

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Real Sociedad Matemática Española

León, Febrero de 2023

Phelps Property U and $C(K)$ spaces

Ch. Cobollo*, A. J. Guirao*, V. Montesinos*

Accordint to R. R. Phelps ([1]), a subspace X of a Banach space Y is said to have *Property U* whenever every continuous linear functional on X has a unique norm-preserving (i.e., Hahn–Banach) extension to Y. Along this talk we introduce the concept of *U-embeddings* between Banach spaces X and Y, that is, isometric embeddings of X into Y whose ranges have property U. Roughly speaking, this perspective also allows us to wonder, given a Banach space X, if it can be thought as a linear subspace with property U of a bigger Banach space Y. In particular, we are interested in the case that $Y = C(K)$, where K is a compact Hausdorff topological space. We develop a systematic study and provide results for general Banach spaces and for some specific set-ups, such as X being a finite-dimensional space or a $C(K)$-space.

Agradecimientos Instituto Universitario de Matemática Pura y Aplicada. Universitat Politècnica de València (Spain). The authors were partially supported by the Universitat Politècnica de València (Spain) and by grant PID2021-122126NB-C33 funded by MCIN/AEI/10.13039/501100011033 and by "ERDF A way of making Europe". The first author was also supported by Generalitat Valenciana (through Project PROMETEU/2021/070 and the predoctoral fellowship CIACIF/2021/378) and by MCIN/AEI/10.13039/501100011033 (through Project PID2019-105011GB). The second author was also supported by Fundación Séneca, Región de Murcia (Grant 19368/PI/14). The third author was also supported by AEI/FEDER (Project MTM2017-83262-C2-1-P of Ministerio de Economía y Competitividad).

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VI Congreso de Jóvenes Investigadores
Real Sociedad Matemática Española
León, Febrero de 2023

Holomorphic functions attaining their weighted norm

Sheldon Dantas,* Rubén Medina†

We present some results on holomorphic functions over a complex Banach space attaining their weighted norm. In particular, we focus on the density for the supremum norm of the set of holomorphic functions attaining their weighted norm in the space of holomorphic functions whose restriction to the unit ball is uniformly continuous. This study shows the relationship between the geometry of the unit ball of a Banach space and the possibility of perturbing holomorphic functions defined over that space to make them attain their weighted norm.

This project has been carried out in collaboration with Sheldon Dantas. See [1] (article).

Acknowledgements The authors are grateful to the Czech Technical University in Prague for giving them the opportunity to develop their research.

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VI Congreso de Jóvenes Investigadores
Real Sociedad Matemática Española
León, Febrero de 2023

Dualidad en retículos de Banach libres

E. García-Sánchez*

Este es un trabajo conjunto con Pedro Tradacete.

El retículo de Banach libre generado por un espacio de Banach E es un retículo de Banach $\mathrm{FBL}[E]$, equipado con un embebimiento lineal isométrico $\phi_E : E \to \mathrm{FBL}[E]$, que verifica la siguiente propiedad universal: para todo retículo de Banach X y todo operador lineal y acotado $T : E \to X$, existe un único homomorfismo de retículos $\hat{T} : \mathrm{FBL}[E] \to X$ tal que $\hat{T}\phi_E = T$, con $\|\hat{T}\| = \|T\|$. Tanto la existencia de $\mathrm{FBL}[E]$ como una construcción explícita del mismo fueron proporcionadas en [1]. Posteriormente [3], dicha construcción se generalizó para dar lugar a un objeto libre que verifica una propiedad universal análoga cuando el retículo de Banach X es p-convexo, el retículo de Banach p-convexo libre generado por el espacio de Banach E, denotado $\mathrm{FBL}^{(p)}[E]$.

Al mismo tiempo, es posible entender el bidual E^{**} de un espacio de Banach E (junto con el embebimiento canónico $J_E : E \to E^{**}$) como el dual libre generado por E, esto es, como un objeto libre sobre E en la categoría de los espacios de Banach duales con operadores adjuntos. Desde este punto de vista, el presente trabajo [2] explora las interacciones existentes entre retículos de Banach libres y duales libres en los siguientes sentidos.

Por una parte, se emplea el principio de reflexividad local para estudiar la relación entre $\mathrm{FBL}^{(p)}[E^{**}]$ (el retículo de Banach p-convexo libre generado por el dual libre sobre E) y $\mathrm{FBL}^{(p)}[E]^{**}$ (el dual libre generado por el retículo de Banach p-convexo libre sobre E), concluyéndose que el primero se embebe canónicamente en el segundo mediante un homomorfismo de retículos isométrico.

Por otra parte, se estudia en qué situaciones es posible combinar estas dos nociones para dar lugar al retículo de Banach dual p-convexo libre generado por un espacio de Banach.

Agradecimientos Este trabajo es parte de la ayuda CEX2019-000904-S-21-3, financiada por MCIN/AEI/10.13039/501100011033 y por el FSE+, y la ayuda CEX2019-000904-S, financiada por MCIN/AEI/10.13039/501100011033.

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Convergence and spaceability within the disc algebra

P.J. Gerlach-Mena*

The pathological behaviour of Fourier series of continuous functions has been broadly studied in the past years. Its origin goes back to du Bois-Reymond (1873), who was the first one to exhibit an example of a continuous function on the unit circle $\mathbb{T} := \{z \in \mathbb{C} : |z| = 1\}$ whose Fourier series diverges at a point.

This was improved by Kahane and Katznelson by extending the divergence to arbitrary sets $E \subset \mathbb{T}$ of (arc length) measure zero, and thus complementing the famous Carleson Theorem on almost everywhere convergence of Fourier series of functions in L^2.

In recent years, these divergence properties have been proved to be topologically generic, that is, not only functions f fulfilling the properties exist, but they hold true for a residual set of functions in the corresponding spaces. Moreover, even linear and algebraic structures can be detected within the set of such functions.

Our aim is to find linear structures in various subsets of the disc algebra. Among others, we consider the set of functions in the disc algebra having a Taylor series about 0 which is unboundedly divergent on a given subset of the unit circle of vanishing arc length measure, and the subsets of functions having uniformly bounded or uniformly convergent Taylor series on the unit circle.

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VI Congreso de Jóvenes Investigadores
Real Sociedad Matemática Española
León, Febrero de 2023

Funciones uniformemente convexas y teorema de Enflo

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Introduciremos la noción de funciones ε-uniformemente convexas y se presentarán sus principales propiedades. Lo más destacable es que la envoltura convexa cerrada de una función ε-uniformemente también lo es. El teorema de Enflo afirma que un espacio superreflexivo admite una norma equivalente uniformemente convexa. No obstante, su demostración es poco entendible de un punto de vista geométrico. En estos espacios mostraremos que la no existencia de arboles diádicos separados de tamaño arbitrariamente grande permite construir funciones uniformemente convexas. Gracias a ello, se expondrá una nueva demostración del teorema de Enflo. También caracterizaremos los conjuntos que admiten una función uniformemente convexa.

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León, Febrero de 2023

El problema del subespacio complementado en retículos de Banach

David de Hevia Rodríguez*

El *problema del subespacio complementado en retículos de Banach* consiste en determinar si todo subespacio complementado en un retículo de Banach es isomorfo a un retículo de Banach. En el artículo [2] se afirma que este problema constituye una de las preguntas (abiertas) más “importantes"de la teoría de retículos de Banach.

En dicho trabajo, también se indica que una de las principales dificultades para abordar esta cuestión está en que “la mayoría" de los criterios que se conocen para descartar que un espacio de Banach pueda ser isomorfo a un retículo de Banach no permiten distinguir entre retículos y subespacio complementados en retículos. Por ejemplo, se sabe que todo espacio de Banach que no posea copias isomórficas de c_0 y ℓ_1 y esté complementado en un retículo es reflexivo, así que el *espacio de James* no puede estar complementado en un retículo. También sabemos que todo subespacio complementado en un retículo de Banach contiene alguna sucesión básica incondicional y, por tanto, el espacio sin sucesiones básicas incondicionales construido por W. T. Gowers y B. Maurey es otro ejemplo de espacio no complementado en un retículo de Banach.

Existen también algunas ramificaciones destacables del problema del subespacio complementado en retículos de Banach, como por ejemplo [1, p. 122]:

- ¿Todo subespacio complementado en $L_1[0,1]$ es isomorfo a ℓ_1o $L_1[0,1]$?
- ¿Todo subespacio complementado en $\mathcal{C}[0,1]$ es isomorfo a un espacio $\mathcal{C}(K)$?

Cabe señalar que una respuesta afirmativa al problema del subespacio complementado en retículos de Banach en el caso separable resolvería las dos preguntas anteriores.

Recientemente, G. Plebanek y A. Salguero Alarcón [4] han construido un espacio de Banach que está 1-complementado en un espacio $C(K)$, pero que no es isomorfo a ningún espacio $C(L)$ (donde L es cualquier compacto Hausdorff), lo que ha resuelto el problema del subespacio complementado en espacios $\mathcal{C}(K)$ en el contexto no separable. G. Martínez Cervantes, A. Salguero Alarcón, P. Tradacete y yo [3] hemos observado que dicho espacio, de hecho, no es isomorfo a ningún retículo de Banach, por lo que también sirve como contraejemplo para el problema del subespacio complementado en retículos. En esta charla explicaré cómo hemos comprobado esta propiedad.

Agradecimientos Quiero dar las gracias especialmente a mi director de tesis, Pedro Tradacete, por su compromiso y disponibilidad constante para guiar mi investigación. Muchas gracias también a Gonzalo y a Luis Carlos por la invitación y por organizar estas sesiones de Análisis Funcional.

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VI Congreso de Jóvenes Investigadores

Real Sociedad Matemática Española

León, Febrero de 2023

La convolución y la propiedad de la diferenciabilidad.

A. Pablo Jiménez Rodríguez*

El operador convolución a menudo se presenta como un operador que toma las propiedades más buenas de las funciones que intervienen, o incluso las mejora. En esta charla examinaremos qué ocurre cuando esta idea se confronta con la propiedad de la diferenciabilidad.

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On the numerical index of the real two-dimensional L_p space

Alicia Quero*

The concept of numerical index was introduced by Lumer in 1968 in the context of the study and classification of operator algebras. This is a constant relating the norm and the numerical range of bounded linear operators on the space. Namely, the numerical index of a Banach space X, $n(X)$, is the greatest constant $k \geq 0$ such that

$$k\|T\| \leq \sup\big\{|x^*(Tx)| \colon x \in S_X,\ x^* \in S_{X^*},\ x^*(x) = 1\big\}$$

for every $T \in \mathcal{L}(X)$.

There are some classical Banach spaces for which the numerical index has been calculated, however the problem of computing the numerical index of L_p-spaces when $p \neq 1, 2, \infty$ has been latent since the beginning of the theory. The aim of this talk is to give an overview of the state of this problem, paying special attention to the two-dimensional real case for which we have recently been able to give a partial solution. More precisely, we have calculated the numerical index of the two-dimensional real L_p-space for $6/5 \leq p \leq 6$ ([1], [2]).

Acknowledgements Research partially supported by projects PGC2018-093794-B-I00 (MCIU/AEI/FEDER, UE), P20-00255 (Junta de Andalucía/FEDER, UE), A-FQM-484-UGR18 (Junta de Andalucía/FEDER, UE), and FQM-185 (Junta de Andalucía/FEDER, UE); and by the Ph.D. scholarship FPU18/03057 (MECD).

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VI Congreso de Jóvenes Investigadores

Real Sociedad Matemática Española

León, Febrero de 2023

Geometría local vs geometría global en la esfera unidad de un espacio de Banach

A. Quilis*

En esta charla hablamos de las propiedades de diferenciabilidad y convexidad de la norma de un espacio de Banach, relacionándolas con la geometría de la esfera unidad. En particular, comparamos las propiedades locales con las globales, y presentamos ejemplos de renormamientos en espacios de Banach ilustrando las diferencias de estos dos tipos de cualidades. Además de comentar resultados clásicos en esta línea, presentamos un método para construir renormamientos con propiedades locales fuertes de diferenciabilidad que no son uniformemente Gâteaux en ninguna dirección. Aplicamos este método a cualquier espacio separable y a $c_0(\Gamma)$ para Γ infinito, consiguiendo además en este último caso propiedades locales adicionales.

Agradecimientos Este proyecto ha sido financiado por PAID-01-19 de la UPV y por la ayuda PID2021-122126NB-C33 de MCIN/AEI/10.13039/501100011033 y "ERDF A way of making Europe".

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León, Febrero de 2023

Banach spaces contained in the set of strongly norm-attaining Lipschitz mappings

Óscar Roldán*

Let M be a pointed metric space (that is, a metric space with a distinguished point 0). Let $\mathrm{Lip}_0(M)$ denote the (Banach) space of lipschitz mappings $f : M \to \mathbb{R}$ such that $f(0) = 0$ endowed with the usual Lipschitz norm. Let $\mathrm{SNA}(M)$ denote the subset of $\mathrm{Lip}_0(M)$ of Lipschitz mappings that *strongly attain their norm*, tthat is, such that there exist two different points $x, y \in M$ such that

$$\frac{|f(y) - f(x)|}{\mathrm{d}(x, y)} = \|f\|.$$

In this talk we will discuss advances on the spaceability of the set $\mathrm{SNA}(M)$ when M is infinite. In [3], it was shown that if M is infinite, then for all $n \in \mathbb{N}$, $\mathrm{SNA}(M)$ isometrically contains ℓ_1^n. The possible sizes of Banach spaces within $\mathrm{SNA}(M)$ was also studied in that paper, as well as the inverse question: the possible size of M if the Banach space Y is inside of $\mathrm{SNA}(M)$. In [1] it was shown that if M is infinite, then $\mathrm{SNA}(M)$ always contains a Banach space that is isomorphic to c_0, answering [3, Questions 1 and 2] in the positive. They left the analogous isometric question open (see [1, Remark 3.6]). Finally, in [2], the previous question was solved: there exist infinite metric spaces M such that $\mathrm{SNA}(M)$ does not contain c_0 isometrically. We will discuss these and some other related results.

Acknowledgements The talk is based on a joint work with Vladimir Kadets, and another joint work with Sheldon Dantas, Rubén Medina, and Andrés Quilis. The author receives economical support from the Spanish Ministerio de Universidades, grant FPU17/02023, and from the research project PID2021-122126NB-C33 / MCIN / AEI / 10.13039 / 501100011033 (FEDER).

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VI Congreso de Jóvenes Investigadores
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León, Febrero de 2023

Sumas torcidas de espacios $c_0(I)$

Alberto Salguero-Alarcón *

Un espacio de Banach Z es *suma torcida* de otros dos espacios de Banach Y y X cuando Z contiene un subespacio isomorfo a Y de manera que Z/Y es isomorfo a X.

El caso más sencillo de sumas torcidas de espacios $C(K)$ son las sumas torcidas de espacios $c_0(I)$. Se conocen algunos ejemplos clásicos, como los espacios de Johnson-Lindenstrauss, y otros más exóticos, como los que se construyen en [2] y [3] con la propiedad de que no son isomorfos a ningún espacio $C(K)$. La idea general es que, si bien una suma torcida de espacios $c_0(I)$ puede no ser isomorfo a un $C(K)$, está “muy cerca” de serlo.

En esta charla describiremos un método general de construcción de sumas torcidas de $c_0(I)$ [1]. Como consecuencia, obtendremos una clasificación de tales espacios, y deduciremos que poseen varias propiedades de interés. El material de esta charla forma parte de un trabajo conjunto con Jesús M. F. Castillo.

Agradecimientos Esta investigación ha sido parcialmente financiada por los proyectos IB20038 de la Junta de Extremadura y PID-2019-103961GB-C21 del Ministerio de Ciencia, Innovación y Universidades.

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VI Congreso de Jóvenes Investigadores
Real Sociedad Matemática Española
León, Febrero de 2023

Derivación de funtores en espacios de Banach

Nazaret Trejo Arroyo*

Quizá nadie exponga mejor que MacLane en [1] el proceso de *derivación de funtores*: *'Un método estándar [de derivación] consiste en: tomar una resolución, aplicar un funtor covariante y tomar homología en el complejo obtenido'*.

En esta charla, en la que confluyen construcciones propias del *álgebra homológica* y conceptos clásicos del análisis funcional, nos centraremos en la derivación de funtores definidos sobre la categoría **Ban**, formada por espacios de Banach y operadores lineales y continuos. Ahora bien, dado que **Ban** no es una *categoría abeliana*, la teoría general falla y derivar un funtor en el contexto del análisis funcional no es tan sencillo como expone MacLane.

Sin embargo, y gracias a ciertas propiedades que poseen los espacios $\ell_1(I)$ y $\ell_\infty(I)$, podremos solventar las dificultades encontradas y adaptar el proceso de derivación para funtores definidos sobre la categoría de los espacios de Banach.

Agradecimientos Este trabajo ha sido financiado por el proyecto IB20038 de la Junta de Extremadura.

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Análisis Armónico y Teoría de Operadores

Organizadores: Luciano Abadías y Marta de León

Miércoles 8 de Febrero
Moderadores: *Israel Rivera y Aingeru Fernández*

10:00 *Fractional discrete Laplacian and (global) unique continuation*, Aingeru Fernández

10:30 *Characterization of the discrete Besov spaces by means of semigroups associated to the discrete Laplacian*, Alejandro Mahillo

11:30 *Weighted estimates on the infinite rooted k-ary tree*, Israel Rivera

12:00 *Resultados en extrapolación para operadores bilineales*, Teresa Luque

12:30 *Extrapolation via Sawyer-type inequalities*, Eduard Roure

13:00 *Extrapolation in quasi-Banach function spaces*, Zoe Nieraeth

Jueves 9 de Febrero
Moderadores: *Adrián González Pérez, Antonio Pérez Hernández, Odí Soler y Tomasz Szarek*

10:00 *The problem of thermal noise in quantum double models*, Antonio Perez Hernández

10:30 *Greedy approximation algorithms for sparse collections*, Guillermo Rey

11:30 *Matrix-valued Calderón-Zygmund theory on noncommutative Hardy spaces*, Antonio Ismael Cano

12:00 *Hilbert transforms for groups acting on tree-like structures*, Adrián González Pérez

12:30 *Group Equivariant Machine Learning*, María Teresa Arias

13:00 *Pointwise ergodic theorems on nilpotent groups*, Tomasz Szarek

17:00 *Fine spectrum of weighted averaging operators of hyperbolic type*, Jesús Oliva

17:30 *Weak spectral decompositions and invariant subspaces*, Miguel Monsalve

18:00 *Finite rank perturbations of normal operators: invariant subspaces and decomposability*, Javier González Doña

Viernes 10 de Febrero
Moderadores: *Miguel Monsalve y Odí Soler*

10:00 *Un modelo funcional en el anillo*, Glenier Bello

10:30 *Characterising multiparameter dyadic BMO spaces with commutators*, Odí Soler

11:30 *Trace of Bloch and BMO(D) functions on Interpolating Sequences*, Alberto Dayan

12:00 *Carleson measure type problem for the tent spaces and integration operators*, Tanausú Aguilar

12:30 *Operators induced by radial measures acting on the Dirichlet space*, Alejandro Más

VI Congreso de Jóvenes Investigadores

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León, Febrero de 2023

Fine spectrum of weighted averaging operators of hyperbolic type

L. Abadías[*] J. E. Galé[†] P.J. Miana [‡] J. Oliva-Maza[§]

Let $\mathcal{J}, \mathfrak{H}$ be the Siskakis' operator [5] and the reduced Hilbert matrix operator respectively, which are given by

$$\mathcal{J}f(z) = \frac{1}{1-z}\int_1^z \frac{f(\xi)}{1+\xi}\,d\xi, \qquad z \in \mathbb{D},$$
$$\mathfrak{H}f(z) = \int_{-1}^1 \frac{f(\xi)}{1-z\xi}d\xi, \qquad z \in \mathbb{D}.$$

In this talk, we provide the spectral picture of two families of multiparameter weighted averaging operators, $\mathcal{J}_\delta^{\mu,\nu}$ and $\mathfrak{H}_\delta^{\mu,\nu}$, which are inspired by operators above. This is done by representing these integral operators via subordination in terms of weighted hyperbolic composition groups $(u_t C_{\varphi_t})$. Such a method reduces the complicated study of the spectrum of $\mathcal{J}_\delta^{\mu,\nu}$ and $\mathfrak{H}_\delta^{\mu,\nu}$ to the simpler study of the spectrum of a differential linear operator of first order, which can be done by similar techniques as in [4].

On the other hand, we prove that under mild assumptions, every cocycle (u_t) of an hyperbolic flow (φ_t) can be written as a coboundary for a weight ω (e.g., $u_t = (\omega \circ \varphi_t)/\omega)$ with zeroes or singularities of polynomial type at the fixed points of (φ_t). This result is key to complete or partly extend some questions about the spectrum of weighted hyperbolic composition operators [1, 2, 3].

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Real Sociedad Matemática Española

León, Febrero de 2023

Carleson measure type problem for the tent spaces and integration operators

Tanausú Aguilar Hernández* Petros Galanopoulos†

In this talk, we deal with a Carleson measure type problem for the tent spaces AT_p^q in the unit disc of the complex plane. These spaces consist of those analytic functions of the tent spaces T_p^q introduced by Coifman, Meyer, and Stein. Well known spaces like Hardy spaces and the Bergman spaces arise as special cases of this family. Let $s, t, p, q \in (0, \infty)$. We will present necessary and sufficient conditions on a positive Borel measure μ of the unit disc in order to exist a constant $C > 0$ such that

$$\int_{\mathbb{T}} \left(\int_{\Gamma_M(\xi)} |f(z)|^t \, d\mu(z) \right)^{s/t} |d\xi| \leq C \|f\|_{T_p^q}^s, \quad f \in AT_p^q,$$

where $\Gamma_M(\xi) = \{z \in \mathbb{D} : |1 - \overline{\xi} z| < M(1 - |z|^2)\}$, $M > 1/2$ and ξ is a boundary point of the unit disc. This problem was originally posed by Luecking.

As a consequence, we apply our results to the study of the boundedness of the integration operators

$$T_g(f)(z) = \int_0^z f(w) g'(w) \, dw,$$

also known as Pommerenke operator, between the tent spaces AT_p^q, for $p, q \in [1, \infty)$. We also consider the boundedness of T_g from a tent space AT_p^q to a Hardy space H^s, where $p, q, s \in [1, \infty)$.

Acknowledgements We are grateful to Professor Manuel D. Contreras and Professor Luis Rodríguez Piazza for their useful comments.

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Group Equivariant Machine Learning

M.T. Arias *

"Symmetry, as wide or as narrow as you may define its meaning, is one idea by which man through the ages has tried to comprehend and create order, beauty, and perfection", said Hermann Weyl. In modern mathematics, symmetry is almost univocally expressed in the language of group theory. In machine learning and its applications to pattern recognition and computer vision, the importance of symmetry has long been recognized. Convolutional neural networks (CNNs) have been extremely successful in the image recognition domain because they ensure equivariance to the group of translations.

The Scattering transforms, introduced by Stéphane Mallat [2] and developed by Bruna and Mallat [3] provide a framework to understand CNNs by replacing trainable filters with multiscale wavelet decompositions, also showcasing the deformation stability and the role of depth in the architecture. Moreover, some of the properties of these transforms rely only on their structure [8]

There are many generalizations of this framework to other domains, including graphs and data lying on manifolds. In general, convolutional networks exhibit robustness to intraclass nuisances when networks are structured such that feature representations are transformed the same way as the input, a desirable property called group equivariance. Given some natural constraints [5], the convolutional structure is not just a sufficient, but also a necessary condition for equivariance to the action of a compact group. In this talk, we present a unified framework for group equivariant networks on homogeneous spaces derived from a Fourier perspective, introduced in [7]. As a particular case, we show how to deal with vector and tensor fields [6], [4].

Our exposition makes use of concepts from representation theory and noncommutative harmonic analysis [9], and from nonlinear approximation theory [1].

Acknowledgements It is a pleasure to thank Davide Barbieri (UAM), Ben Blum-Smith (JHU), Wilson Gregory (JHU), Eugenio Hernández (UAM), David W. Hogg (NYU) and Soledad Villar (JHU) for valuable discussions and input.

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VI Congreso de Jóvenes Investigadores

Real Sociedad Matemática Española

León, Febrero de 2023

Un modelo funcional en el anillo

G. Bello*

En esta charla presentaremos un modelo funcional unitariamente equivalente para operadores invertibles T en espacios de Hilbert satisfaciendo

$$-T^{*2}T^2 + (1 + r^2)T^*T - r^2 I \geq 0,$$

donde $0 < r < 1$. Dicha expresión proviene de aplicar el cálculo funcional hereditario de Agler al polinomio $(1 - t)(t - r^2)$. También discutiremos la relación de nuestro modelo con el de Sz.-Nagy–Foias para contracciones en espacios de Hilbert. Como consecuencia, probaremos que el anillo unidad cerrado $r \leq |z| \leq 1$ es un conjunto K-espectral completo para T. La charla se basará en un trabajo en colaboración con D. Yakubovich.

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Matrix-valued Calderón-Zygmund theory on noncommutative Hardy spaces

A.I. Cano Mármol[*] É. Ricard[†]

Given a von Neumann algebra $\mathcal{M}$, any essentially bounded function $f : \mathbb{R}^n \to \mathcal{M}$ can be understood as an operator belonging to the tensor product von Neumann algebra $L_\infty(\mathbb{R}^n)\overline{\otimes}\mathcal{M}$. This approach enables us to formulate an appropriate analogous for Hardy spaces of matrix-valued functions defined on $\mathbb{R}^n$ [2]. In this talk, we will introduce a new atomic decomposition for these spaces [1], and review some consequences on noncommutative Calderón-Zygmund theory on operator-valued Hardy spaces. This is joint work with Éric Ricard.

Agradecimientos The first author was supported by Grant SEV-2015-0554-19-3 funded by MCIN/AEI/10.13039/50110 0011033, the Madrid Government Program V PRICIT Grant SI1/PJI/2019-00514, and the Spanish Grant PID2019-107914GB-I00 'Fronteras del Análisis Armónico' (MCIN / PI J. Parcet) as well as Severo Ochoa Grant CEX2019-000904-S (ICMAT), funded by MCIN/AEI 10.13039/501100011033.

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León, Febrero de 2023

Trace of Bloch and $BMO(\mathbb{D})$ functions on Interpolating Sequences

Alberto Dayan [*], Adrian Llinares [†], Karl-Mikael Perfekt [‡]

In [4], Sundberg characterized the values that a $BMOA$ function can attain on a interpolating sequence in the unit disc. Motivated by his work, in this talk we will consider the problem of characterizing the trace of $BMO(\mathbb{D})$ functions on interpolating sequences for the Bergmann and the Hardy space. In order to do so, we will use a John-Nieremberg type inequality for the class $BMO(\mathbb{D})$ recently proved by Nicolau and Linami in [2]. After restricting our attention to the subclass of those $BMO(\mathbb{D})$ functions which are quasi-Lipschitz with respect the hypebolic distance, we will then solve an interpolation problem by first considering its dyadic version in the class $BMO(\mathbb{D})_d$ for which the BMO condition is checked only on Carleson squares arising from dyadic intervals in the unit circle, and by then proving that the average of such dyadic solutions still is in $BMO(\mathbb{D})$ and has the desiderd Lipschitz properties. Such extra Lipschitz assumptions are quite natural, (see [2] and [3]), and they play a key role in extending the avereging argument in [1] to our $BMO(\mathbb{D})$ problem.
We will also discuss some connection with the analogous problem for the class of Bloch functions, and with some factorization and extension properties of Bekollé-Bonami weights of bounded hyperbolic oscillation on the unit disc.

This talk is based on a joint work with Adrian Llinares and Karl-Mikael Perfekt.

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VI Congreso de Jóvenes Investigadores
Real Sociedad Matemática Española
León, Febrero de 2023

Fractional discrete Laplacian and (global) unique continuation

A. Fernández-Bertolin * L. Roncal† A. Rüland ‡

We say that an operator P enjoys the weak unique continuation property (WUC) if no solution of $Pu = 0$, besides possibly the zero function, vanishes on an open set. The first operator one considers is $-(\Delta + V)$, which, under some conditions on the potential V, satisfies WUC. As for fractional powers of the Laplacian, Ghosh, Salo and Uhlmann showed in [3] that they also satisfy some sort of global WUC: If $\Omega \subset \mathbb{R}^d$ is an open set and $u \in H^r(\mathbb{R}^d)$, for some $r \in \mathbb{R}$ satisfies

$$u = 0 = (-\Delta)^s u \text{ in } \Omega,$$

then $u \equiv 0$ in $\mathbb{R}^d$. Based on our preprint [1], the motivation of this talk is to study the failure of these properties in the discrete case. However, since WUC holds in the continuum and our operators are approximations of those from the continuum, one should be able to recover in some sense the unique continuation properties. With this in mind, we will give quantitative versions of unique continuation for the square root of the discrete Laplacian, based on three balls inequalities related to the one given by the authors in collaboration with D. Stan (Cantabria) to study quantitative unique continuation for discrete magnetic Schrödinger operators [2].

Acknowledgments. We are indebted to D. Stan (Cantabria) for helpful discussions regarding this project.

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VI Congreso de Jóvenes Investigadores

Real Sociedad Matemática Española

León, Febrero de 2023

Operators induced by radial measures acting on the Dirichlet space

Petros Galanopoulos [*] Daniel Girela [**] Alejandro Mas [**] Noel Merchán [***]

Let $\mathbb{D}$ be the unit disc in the complex plane. Given a positive finite Borel measure μ on the radius $[0,1)$, we let μ_n denote the n-th moment of μ and we deal with the action on spaces of analytic functions in $\mathbb{D}$ of the operator of Hibert-type $\mathcal{H}_\mu$ and the operator of Cesàro-type $\mathcal{C}_\mu$ which are defined as follows:

If f is holomorphic in $\mathbb{D}$, $f(z) = \sum_{n=0}^{\infty} a_n z^n$ $(z \in \mathbb{D})$, then $\mathcal{H}_\mu(f)$ is formally defined by

$$\mathcal{H}_\mu(f)(z) = \sum_{n=0}^{\infty} \left(\sum_{k=0}^{\infty} \mu_{n+k} a_k \right) z^n, \quad z \in \mathbb{D}$$

and $\mathcal{C}_\mu(f)$ is defined by

$$\mathcal{C}_\mu(f)(z) = \sum_{n=0}^{\infty} \mu_n \left(\sum_{k=0}^{n} a_k \right) z^n, \quad z \in \mathbb{D}.$$

These are natural generalizations of the classical Hilbert and Cesàro operators. A good amount of work has been devoted recently to study the action of these operators on distinct spaces of analytic functions in $\mathbb{D}$, see for example [1] and [2]. In this talk we will present some results about the action of the operators $\mathcal{H}_\mu$ and $\mathcal{C}_\mu$ on the Dirichlet space $\mathcal{D}$ and, more generally, on the analytic Besov spaces B^p $(1 \leq p < \infty)$.

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Finite rank perturbations of normal operators: invariant subspaces and decomposability

F. Javier González-Doña[*]

In this talk, we study the Invariant Subspace Problem for compact perturbations of diagonalizable normal operators acting boundedly on infinite-dimensional, separable, complex Hilbert spaces.

In particular, we will present results regarding the existence of non-trivial closed hyperinvariant subspaces for for finite rank perturbations of diagonalizable, normal operators extending previous results of Foias, Jung, Ko and Pearcy and Fang and Xia. We will address the main techniques involved in the proof, based on Local Spectral Theory.

Finally, we will introduce the concept of spectral decomposability of an operator and show that a large class of such operators are decomposable, having, consequently, a rich lattice of invariant subspaces.

This is part of joint works with Eva A. Gallardo-Gutiérrez [1, 2, 3].

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VI Congreso de Jóvenes Investigadores

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León, Febrero de 2023

Hilbert transforms for groups acting on tree-like structures

A. González Pérez [*], J. Parcet [†], R. Xia [‡]

The Hilbert transform in $\mathbb{R}$ is a classical example of a Fourier multiplier. Its boundedness on the L^p spaces was proved by M. Riesz. Later on, Cotlar [1] proved its L^p-boundedness in the 50s using a recurrence relation, now known as the Cotlar identity, to circunvent the need for singular integral techniques. Let H be the Hilbert transform. Cotlar's identity states that

$$H\, f\, \overline{H\, f} \;=\; H(f\, \overline{H\, f}) + \overline{H(f\, \overline{H\, f})} - H\big(\overline{H(f\, \overline{f})}\big).$$

More recently, Mei and Ricard [2] realized the usefulness of the Cotlar identity to prove the L^p-boundedeness of operators in settings in which kernel estimates are unavailable. Doing so, they successfully proved the L^p-boundedness of Hilbert transform analogues over free groups. In this talk, we will present a Cotlar identity for general groups and provide a model of Hilbert transforms satisfying it coming from actions on $\mathbb{R}$-trees. Such a model provides non-trivially bounded transforms when the action doesn't have fixed points. Examples of such groups include amalgamated free products, totally ordered groups and HNN extensions among others.

This is a joint work with Javier Parcet and Runlian Xia.

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VI Congreso de Jóvenes Investigadores
Real Sociedad Matemática Española
León, Febrero de 2023

The problem of thermal noise in quantum double models

Angelo Lucia* David Pérez-García† Antonio Pérez-Hernández.‡

One of the main difficulties in the construction of a quantum computer is dealing with *decoherence*, the process by which the information stored in a quantum system is lost due to the interaction between the system and the environment. For instance, since the system cannot be completely isolated, the interaction with the environment causes that the system increases its temperature, which makes that probabilistic errors continually occur while the computation process is being performed (*thermal noise*).

In the 90s, the development of the area of *quantum error correction* fostered the idea that it may be possible to overcome this problem. Introducing certain error-correction protocols, one could protect the stored information during enough time to perform all the computations. Nevertheless, this approach is not exent of other technical issues to be solved.

Motivated by the classical case, an alternative idea was trying to find systems that acted as *self-correcting quantum memories*, namely systems that protect quantum information against thermal errors (for all temperatures below a threshold) without the need of active error correction. Amongst the candidates were Kitaev's quantum double models [4]. Nowadays, the general belief is, however, that in 2D there are no self-correcting quantum memories. In order to formally solve the problem, one has to consider the thermal evolution operator modeled by the Davies master equation. Self-correction would not be possible if the noise operator relaxes fast to the Gibbs ensemble, where all information is lost. The key quantity that controls this relaxation time is the spectral gap of the Davies Lindbladian generator, so that having a gap independent of the system size excludes self-correction. This result was proven by Alicki et al. in [1] for the toric code, and later extended to the family of Abelian quantum double models by Kómár et al. [2]. In a recent work [3], we have finally proved the statement for all quantum double models (not necessarily Abelian) using different techniques coming from the area of Tensor Networks.

In this talk, we will introduce the mathematical framework that lies behind this problem and its solution.

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VI Congreso de Jóvenes Investigadores

Real Sociedad Matemática Española

León, Febrero de 2023

Resultados en extrapolación para operadores bilineales

T .Luque*

Siguiendo la argumentación de extrapolación de Rubio de Francia presentada en [2] para deducir la acotación de operadores en el extremo a partir de su acotación en espacios de Lebesgue con pesos, estudiamos su utilidad y generalización en el contexto bilineal, en concreto para multiplicadores bilineales de Fourier.

La extrapolación multilineal es un tema de gran actualidad que ha sido muy analizado desde distintos puntos de vista desde la publicación inicial [4]. Algunas de las líneas de argumentación que mencionaremos están recogidas en [5] y [3].

Parte de los resultados que discutiremos en la charla han sido publicados en [1].

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VI Congreso de Jóvenes Investigadores
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León, Febrero de 2023

Characterization of the discrete Besov spaces by means of semigroups associated to the discrete Laplacian

Alejandro Mahillo *

The starting point of our work is the papers published in the late 1960s by Stein and Taibleson where Lipschitz (or Hölder) and Besov functions are characterized using certain types of integral estimates of the Gauss semigroup, $\{e^{t\Delta_d}\}_{t>0}$, and of the Poisson semigroup $\{e^{-y\sqrt{-\Delta_d}}\}_{y>0}$. This type of results are beneficial since they allow us to obtain regularity results on the semigroup in a much more direct way, without going through the computation of pointwise expression. This makes us question what happens with other types of "laplacians", as for example in the discrete case of these spaces.

In this context, a first article by Abadías and De León-Contreras [3] is framed where they characterize the discrete Hölder spaces by means of the heat semigroup and Poisson semigroup associated to the discrete Laplacian. It then naturally arises to wonder whether the results obtained by Stein and Taibleson for Besov spaces are also verified for the discrete case.

In this paper we give an affirmative answer to that question by characterizing the discrete Besov spaces using the heat semigroup and Poisson semigroup associated to the discrete Laplacian. In order to do so, it is necessary to compute certain bounds on the discrete heat kernel which we believe to be relevant in themselves. Finally, we give some applications of these results to the computation of some relevant finite differences operators.

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VI Congreso de Jóvenes Investigadores

Real Sociedad Matemática Española

León, Febrero de 2023

Weak spectral decompositions and invariant subspaces

Miguel Monsalve López*

One of the most successful strategies to produce invariant subspaces for a continuous linear operator $T : X \to X$ (acting on a complex Banach space X) is to split its spectrum $\sigma(T)$ into suitable pieces. A cornerstone at this regard is given by the class of normal operators acting on Hilbert spaces. In fact, along the years, several attempts have been pursued to transfer some of the most profitable spectral properties of normal operators to the Banach space setting. In this line, the class of decomposable operators, introduced by Foiaş in the sixties, plays an essential role in local spectral theory.

A common technique to obtain spectral decompositions for an operator T relies on the construction of a functional calculus which relates subsets of $\sigma(T)$ with the zero set of complex-valued functions. Following those ideas, in our article [4], we introduce a weaker functional calculus to provide invariant subspaces for large classes of Banach space operators via local spectral manifolds. Our approach uses abstract harmonic analysis along with local spectral theory to construct non-trivial spectral decompositions. In particular, we generalize previous results on the existence of invariant subspaces due to Atzmon [1], Beauzamy [2] and Colojoară and Foiaş [3], to general classes of Banach algebras with partitions of unity. As a concrete application in the context of Beurling algebras, we show that certain Bishop operators (one of the most promising potential counterexamples to the Invariant Subspace Problem) satisfy a weak type of decomposability.

Agradecimientos: This is a joint work with E. A. Gallardo-Gutiérrez.

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Extrapolation in quasi-Banach function spaces

Z. Nieraeth*

Rubio de Francia's extrapolation theorem allows one to show that an operator that is bounded on weighted Lebesgue spaces for a single exponent and with respect to all weights in the associated Muckenhoupt class has to also be bounded for every exponent. As a matter of fact, in the previous years it has been shown that the operator has to be bounded on a much larger class of spaces, including Lorentz, variable Lebesgue, and Morrey spaces, and further weighted Banach function spaces. In this talk I will discuss a recently obtained unification and extension of some of these results by presenting an extrapolation theorem for general Banach function spaces. I will also discuss limited range and off-diagonal variants in the setting of quasi-Banach function spaces.

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Greedy approximation algorithms for sparse collections

G. Rey*

We describe a greedy algorithm that approximates the Carleson constant of a collection of general sets. The approximation has a logarithmic loss in a general setting, but is optimal up to a constant with only mild geometric assumptions. The constructive nature of the algorithm gives additional information about the almost-disjoint structure of sparse collections.

As applications, we give three results for collections of axis-parallel rectangles in every dimension. The first is a constructive proof of the equivalence between Carleson and sparse collections, first shown by Hänninen. The second is a structure theorem proving that every finite collection $\mathcal{E}$ can be partitioned into $\mathcal{O}(N)$ sparse subfamilies where N is the Carleson constant of $\mathcal{E}$. We also give examples showing that such a decomposition is impossible when the geometric assumptions are dropped. The third application is a characterization of the Carleson constant involving only $L^{1,\infty}$ estimates.

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Weighted estimates on the infinite rooted k-ary tree

Israel P. Rivera-Ríos*

In this talk we present weighted endpoint estimates for the Hardy-Littlewood maximal function on the infinite rooted k-ary tree. In particular we present the following Fefferman-Stein estimate

$$w(x \in T : Mf(x) > \lambda) \leq c_s \frac{1}{\lambda} |f(x)| M(w^s)(x)^{\frac{1}{s}} dx \qquad s > 1$$

which turns out to be sharp, in the sense that it does not hold in general if s = 1. This result is a generalization of the unweighted case ($w \equiv 1$) independently obtained by Naor-Tao [2] and Cowling-Meda-Setti [1].

We will also present sufficient conditions for the strong type estimates in the case $p > 1$ for which the usual A_p condition does not provide useful information.

This talk is based in joint works with Sheldy Ombrosi and Martín Safe [4, 3].

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Extrapolation via Sawyer-type inequalities

Eduard Roure Perdices*

We present a multi-variable extension of the restricted weak-type extrapolation theory of Rubio de Francia. Our approach does not involve the so-called Rubio de Francia's iteration algorithm; instead, we rely on the following Sawyer-type inequality for the weighted Hardy-Littlewood maximal operator:

$$\left\| \frac{M_u(fv)}{v} \right\|_{L^{1,\infty}(uv)} \leq C_{u,v} \|f\|_{L^1(uv)}, \quad u, uv \in A_\infty.$$

Acknowledgements The author is supported by a Margarita Salas grant [University of Barcelona / Spanish Ministry of Universities / Spain's Recovery, Transformation and Resilience Plan / European Union NextGenerationEU].

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VI Congreso de Jóvenes Investigadores
Real Sociedad Matemática Española
León, Febrero de 2023

Characterising multiparameter dyadic BMO spaces with commutators

O. Soler i Gibert*

In this talk we will discuss multiparameter extensions of the space BMO. First, we will review some properties of the classical BMO space. In particular, we will focus on commutators of the form $[H, b]$, where b stands for multiplication by function b in BMO and H is the Hilbert transform. The interest on such commutators comes from Nehari's Theorem, which states that the norm of the commutator $[H, b]$ as an operator from L^2 to itself is comparable to the BMO norm of b. Then, we will talk about the corresponding versions of Nehari's Theorem in various generalisations of BMO : weighted spaces and multiparameter spaces. Lastly, we will explain how to get some of these results for the multiparameter dyadic BMO spaces.

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León, Febrero de 2023

Pointwise ergodic theorems on nilpotent groups

T.Z. Szarek*

The famous Furstenberg–Bergelson–Leibman conjecture says that the ergodic averages converge pointwise almost everywhere provided that the underlying measure preserving transformations generate a nilpotent group. The purpose of this talk is to discuss recent progress towards solving this conjecture. The talk will be based on joint work with Alexandru Ionescu, Akos Magyar and Mariusz Mirek [1].

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Sistemas Dinámicos y Ecuaciones Diferenciales

Organizadores: Cristina Lois y José Manuel Uzal

Jueves 9 de Febrero
Moderadores: *Cristina Lois Prados y José Manuel Uzal Couselo*

10:00 *Attractor for non-autonomous Lotka-Volterra cooperative systems*, Juan García-Fuentes

10:30 *Contributions to the classification of the global dynamics of polynomial differential systems*, Érika Diz-Pita

11:30 *Tipping phenomena on nonautonomous single-species population models subject to Allee effect*, Jesús Dueñas

12:00 *Planar Hamiltonian systems: Index theory and applications*, Eduardo Muñoz-Hernández

12:30 *Applications for L_3 in celestial mechanics: from mathematical modelling to real-system comparisons*, Begoña Nicolás

13:00 *The Hydrogen atom in a circularly polarized microwave field: Trajectories from and to the nucleus*, Óscar Rodríguez

Viernes 10 de Febrero
Moderadores: *Cristina Lois Prados y José Manuel Uzal Couselo*

10:00 *Bursting neurons: bifurcation structure of spike-adding processes in the Hindmarsh-Rose model*, Lucía Pérez

10:30 *Invariant manifolds of vector fields with parabolic invariant tori*, Clara Cufí-Cabré

11:30 *Structure of attractors for a nonlocal Chafee-Infante problem*, Rubén Caballero

12:00 *A link between difference equations and delay differential equations*, Sebastián Buedo-Fernández

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León, Febrero de 2023

A link between difference equations and delay differential equations

A. Sebastián Buedo-Fernández*

One of the main aims of the PhD thesis of the author [2] was to study the qualitative behaviour of the solutions of the following type of scalar delay differential equations (DDEs):

$$x'(t) = -x(t) + f(x(t-\tau)), \tag{1}$$

where $\tau > 0$ and $f : \mathbb{R} \to \mathbb{R}$ is a continuous function. The former kind of differential equations, also known as Mackey-Glass DDEs, finds applications in phenomena related to the destruction and the delayed production for a certain magnitude, such as in the field of population dynamics.

The solutions of (1) have some features that do not appear in classical ordinary differential equations in $\mathbb{R}^n$, since its natural phase space is infinite-dimensional. Hence, one may correctly expect that, in general, the qualitative study of (1) is hard.

However, some decades ago, the works of an der Heiden and Mackey [1] and Mallet-Paret and Nussbaum [4] showed how to extract relevant information about the global attractivity of equilibria of (1) by simply studying the asymptotic properties of the solutions of the scalar difference equation

$$x_{n+1} = f(x_n). \tag{2}$$

Thus, one can use many known results for (2) in order to simplify the qualitative analysis of (1).

In this talk, we will explain such a link between discrete-time and continuous-time dynamics in terms of the asymptotic properties and how to extend it to more general cases such as systems of equations, following the ideas of Liz and Ruiz-Herrera [3].

Acknowledgements The author would like to thank his PhD supervisors, Eduardo Liz Marzán and Rosana Rodríguez López, for their remarkable support over the last years, and Daniel Cao Labora and many other colleagues for all those shared ideas and discussions that have been so helpful.
Moreover, he wants to acknowledge former financial support from Government of Spain (FPU16/04416), Xunta de Galicia (ED481A-2017/030) and the research group Nonlinear Differential Equations [GI-1561] (USC).

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León, Febrero de 2023

Structure of attractors for a nonlocal Chafee-Infante problem

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If we consider the non-local equation

$$\frac{\partial u}{\partial t} - a(\|u\|^2_{H^1_0})\frac{\partial^2 u}{\partial x^2} = \lambda f(u) \tag{1}$$

with Dirichlet boundary conditions, then it is possible to define a suitable Lyapunov functional. In [1] it is shown that regular and strong solutions generate (possibly) multivalued semiflows having a global attractor which is described by the unstable set of the stationary points.

In the case where the function f is odd and equation (1) generates a continuous semigroup, the existence of fixed points of the type given in the Chafee-Infante problem was established in [2].

Moreover, if a is non-decreasing, then they coincide with the ones in the Chafee-Infante problem. In this work we extend these results for a more general function f.

Our goal is to describe the structure of the attractor as accurately as possible. For this aim, we focus on the particular situation where the domain is one-dimensional and the function f is of the type of the standard Chafee-Infante problem, for which the dynamics inside the attractor has been completely understood [3].

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León, Febrero de 2023

Invariant manifolds of vector fields with parabolic invariant tori

A. Clara Cufí-Cabré*   B. Ernest Fontich†

We consider analytic vector fields having an invariant parabolic torus with nilpotent part, $\mathcal{T}$. By *parabolic torus with nilpotent part* we mean that the dynamics in the normal directions of $\mathcal{T}$ is parabolic and that the differential of the system restricted to that directions is given by a non-diagonalizable matrix. Those vector fields may also depend quasiperiodically on time. In this context, we study the existence and regularity of invariant manifolds asymptotic to $\mathcal{T}$ and we show that they are analytic away from $\mathcal{T}$, using the parameterization method for invariant manifolds [2]. We also provide an algorithm to compute an approximation of a parameterization of the invariant manifolds, which also depends quasiperiodically on time, and a normal form of the restricted dynamics on them. We present an application of our result to the study of the scattering of helium atoms off copper corrugated surfaces. This is work generalizes the results for maps presented in [1].

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León, Febrero de 2023

Contributions to the classification of the global dynamics of polynomial differential systems

A. Érika Diz-Pita*

Polynomial differential systems have been used for modeling many real problems from different areas, such as the evolution of coexisting species in biology, the process of some chemical reactions, the evolution of some kind of stars and blackholes, and also economic and social problems.

In general the explicit solutions of differential systems can not be obtained, so the qualitative study of these systems is particularly important because of the information it can provide on the behavior and evolution of the systems.

In this talk we will discuss the results obtained for some families of polynomial systems for which we have studied and classified their global dynamics, and we will explain the used techniques.

Among other things, we will present the complete classification obtained in the works [1, 2, 3, 4] for a planar Kolmogorov family. This family has been obtained from a Lotka-Volterra system on dimension three which has a large number of parameters, so it was difficult to perform a complete study of it directly.

On the other hand, we will present some other systems with a more applied motivation, such as the one studied in [5].

Acknowledgements This work has been partially supported by the Ministerio de Ciencia e Innovación, Agencia Estatal de Investigación (Spain), grant PID2020-115155GB-I00 and the Consellería de Educación, Universidade e Formación Profesional (Xunta de Galicia), grant ED431C 2019/10 with FEDER funds.

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León, Febrero de 2023

Tipping phenomena on nonautonomous single-species population models subject to Allee effect

A. Jesús Dueñas* B. Carmen Núñez* C. Rafael Obaya*

In this talk, we study the transition between two nonautonomous coercive scalar ordinary differential equations given by a function with concave derivative. The classification of the transition equation into three cases, depending on the existence and behavior of hyperbolic solutions, allows us to describe all the possible dynamical scenarios. The skew-product formulation is used to obtain some of the relevant results. Tipping is described as a loss of hyperbolic solutions, and some useful criteria determining the dynamical case of a given equation are found. Since equations with concave derivative are found to be suitable models for single-species populations subject to Allee effect (that is, subject to a positive correlation between the size of a population and its fitness), the previous results are applied in models of mathematical biology. Rate-induced tracking phenomena are found to be responsible for the extinction of a native species and for the invasion of a habitat of a non-native one in some numerical examples. Other numerical simulation relates the strength of Allee effect with the possibility of a critical extinction.

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Atractor for non-autonomous Lotka-Volterra cooperative systems

A. Juan García-Fuentes* B. Piotr Kalita† C. José A. Langa* D. Antonio Suárez*

The autonomous Lotka-Volterra model is used to study the evolution of species population from an ecosystem. It is well known the conditions that the model has to hold to obtain the existence of an stable equilibrium point such that all species survive (permanence solution), or on the contrary, some species go to extinction [4]. Once it is known the stable point, it is possible to understand and build all the structure of the attractor of the system with the heteroclinic connections between the unstable equilibrium points and the stable one.

Passing these results to the non-autonomous situation, i.e. when the parameters are depending on time, is non-trivial. Lazer and Ahmad studied the Lotka-Volterra non-autonomous model in the case that the species compete against them. They show conditions that the system has to hold to obtain an stable solution, first in case that there is permanence [1], to see later the extinction [2]. On the other hand, Redheffer found other conditions to obtain from non-autonomous Lotka-Volterra model with general interactions between species the existence of an stable solution, but only when there is permanence [3].

In this talk we are going to consider the non-autonomous Lotka-Volterra system where the species cooperate between them all time. We are going to show conditions to obtain the existence of a stable solution, in both the permanence and extinction situations. Once we have obtained these stable solutions, we are going to prove how to create the heteroclinic connections between the stable solution and the semi-stables ones, and thus obtain the whole structure of the attractor yielded from a non-autonomous ODE.

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León, Febrero de 2023

Planar Hamiltonian systems: Index theory and applications

A. Eduardo Muñoz-Hernández*

In this communication, we will analyze a planar Hamiltonian system of the type

$$Jz' = \nabla_z H(t, z),$$

where $H : \mathbb{R} \times \mathbb{R}^2 \to \mathbb{R}$ is a function periodic in the time variable, such that $\nabla_z H(t, 0) \equiv 0$ and $\nabla_z H(t, z)$ is asymptotically linear for $|z| \to +\infty$.

First, it will be presented a brief recap of the index theory for linear planar Hamiltonian systems with periodic coefficients providing the relationship between the stability of the system, the Conley-Zehnder index, the winding number of solutions and their respective asymptotic indices. Although most of these relationships are well known, we take a slightly different a priori classification in order to give a more natural definition of the Conley-Zehnder index for resonant cases from the point of view of the rotational behavior of solutions. This seems to be more appropriate to the analysis of the existence and multiplicity of subharmonics in nonlinear planar Hamiltonian systems.

In the second part of the talk, by using the Poincaré-Birkhoff fixed point theorem, it will be shown that the above nonlinear system has subharmonic solutions of any order k large enough, whenever the rotation numbers (or, equivalently, the mean Conley-Zehnder indices) of the linearizations of the system at zero and at infinity are different. Different applications are given to the case of planar Hamiltonian systems coming from second order scalar ODEs.

This is a joint work with professor Alberto Boscaggin from Università degli Studi di Torino.

Acknowledgements The author wants to thank the support of the Research Project PID2021-123343NB-I00 of Ministerio de Ciencia e Innovación and the predoctoral contract CT42/18-CT43/18 of Universidad Complutense de Madrid

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Applications for L_3 in celestial mechanics: from mathematical modelling to real-system comparisons

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Bicircular Problem (BCP) is a modification of the Restricted Three Body Problem (RTBP) in which the gravitational effect of a third massive body is introduced as a time-periodic perturbation. This makes the BCP to be a non-autonomous system in which energy is not conserved. Consequently, trajectories in a system described by the BCP are not bounded to any particular region.

Five equilibrium points are known to be found on the RTBP. Under generic non-degeneracy and non-resonance conditions, these points become periodic orbits when the time-periodic perturbation of the BCP is added. As a consequence, the families of periodic orbits around the Lagrangian points in the RTBP become Cantorian families of two-dimensional invariant tori in the BCP.

In our analysis we use the planar BCP to describe the movement of a particle of negligible mass, such as an asteroid, subjected to the gravitational attraction of the Earth, the Moon and the Sun; being the Sun the perturbing body. In particular, we focus on the less studied of the Lagrangian points, L_3. We numerically compute the quasi-periodic solutions around L_3 and the related stable and unstable invariant manifolds.

Recent work about the role that this family plays in the solar perturbed Earth-Moon system shows how the invariant manifolds of the hyperbolic invariant tori in the neighbourhood of L_3 describe trajectories connecting the vicinity of Earth and the Moon, or the inside and outside system, [1]. To present so, we discretise the fundamental domain (FD) of the invariant manifolds as two dimensional colour maps, coloured according to the faith of the invariant manifolds. This makes clear that L_3 family of invariant tori supposes a rich mechanism of transport to describe different phenomena, for example the travel of lunar ejecta towards the Earth, that was checked in a more realistic model, based on JPL ephemeris.

Additionally, we show how the invariant manifolds can be used to capture a NEA (Near Earth Asteroid). For this application it is necessary to compute an accurate approximation of the invariant manifolds. For this we have implemented a high order approximation of the invariant manifolds of invariant tori, based on the parametrization method combined with the jet transport technique, [2].

Our goal in this talk is to emphasise the importance of mathematical modelling of real-world phenomena and the necessity of checking the theoretical results obtained through simplified mathematical models with more reliable ones whenever is possible.

Acknowledgements Authors thank national project PID2021-125535NB-I00 for its financial support.

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Bursting neurons: bifurcation structure of spike-adding processes in the Hindmarsh-Rose model

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Mathematical neuroscience has a notable role nowadays. Understanding a system as complex as the brain requires to achieve a deeper understanding of the dynamics of neurons, its basic units. One of the behaviours often displayed by neurons is bursting. Bursting consists of alternating periods of activity (trains of action potentials, also known as spikes) with periods of rest. When a parameter of a neuron model changes its value, it is possible that a gaining of one spike in the bursting trains occurs. This process of gaining an extra spike is known as spike-adding. We investigate the different spike-adding processes that can be found in the Hindmarsh-Rose neuron model, which is widely accepted as a prototype for fold/hom and fold/Hopf burstings. There are two different mechanisms for the fold/hom case (continuous and chaotic) and a single (and continuous) mechanism for the fold/Hopf case. While all these processes are presented, to some extent, in the literature, our study gives a unified vision that allows to construct a spike-adding map determined by the bifurcation structure. Moreover, our results allow to understand the dynamics exhibited when borders are crossed, a crucial point not previously studied.

Acknowledgements Financial support by the Spanish Research Projects MTM2017-87697-P, PID2020-113052GB-I00 and PID2021-122961NB-I0 is gratefully acknowledged.

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The Hydrogen atom in a circularly polarized microwave field: Trajectories from and to the nucleus

A. Esther Barrabés* B. Mercè Ollé† C. Óscar Rodríguez†

In this talk, we will study the problem of the hydrogen atom submitted to a circularly polarized microwave field. This problem can be modelled as a perturbed Kepler problem (see [1]) and its Hamiltonian formulation is given by

$$H = \frac{1}{2}(p_x^2 + p_y^2 + p_z^2) - \frac{1}{r} + F(x\cos\omega t + y\sin\omega t),$$

where (x, y, z) and (p_x, p_y, p_z) are the canonical coordinates and their conjugate momenta, $r^2 = x^2 + y^2 + z^2$, ω is the angular frequency of the microwave field, t is the time and $F > 0$ is the field strength. After a suitable change of coordinates and time we can express the problem with a Hamiltonian with two degrees of freedom, that depends on a single parameter $K > 0$ and is expressed as a simple perturbation of the well known Kepler problem. Despite its simplicity, this problem has a very complex dynamics (see [2]).

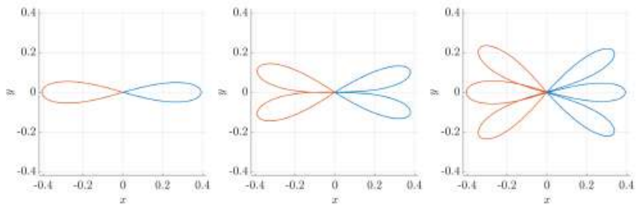

Figure 1: The two n-ECOs with $n = 1, 2, 3$ for $C = 5$ and $K = 0.1$.

We will restrict to the planar case and using the Levi-Civita regularization we will focus on the so called n-ejection-collision orbits (n-EC orbits). These orbits eject from the nucleus (located at the origin), describe n maximum distance passages from the origin and finally collide with it. On the analytical side, we claim and prove the existence of two families of n-EC orbits for big enough ranges of the energy following the ideas introduced in [3] and numerically we will study the evolution of these families.

We will analyze the bifurcations that appear when doing the continuation of the families for $K > 0$ and n fixed. We will see that for smaller values of C there exist more than two n-EC orbits. We will discuss some particularities of the bifurcating families, which have two branches, and we will describe the properties concerning periodicity and symmetry of their n-EC orbits.

Acknowledgements This work is supported by the Spanish State Research Agency, through the Severo Ochoa and María de Maeztu Program for Centers and Units of Excellence in R&D (CEX2020-001084-M) and by the Spanish MINECO/FEDER grant PID2021-123968NB-I00 (AEI/FEDER/UE).

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Aspectos Computacionales y Aplicaciones de las Ecuaciones en Derivadas Parciales Hiperbólicas

Organizadores: Celia Caballero-Cárdenas, Irene Gómez-Bueno y Ernesto Pimentel García

Jueves 9 de Febrero
Moderadores: *Celia Caballero Cárdenas y Ernesto Pimentel García*

11:30 *High-order Implicit-Explicit scheme for the Exner model without gravity effects*, Emanuele Macca

11:50 *Exactly well-balanced schemes for the shallow-water equations with Coriolis forces*, Víctor González Tabernero

12:10 *Una familia de esquemas de volúmenes finitos implícitos y semi-implícitos de alto orden bien equilibrados para sistemas de leyes de equilibrio unidimensionales*, Irene Gómez Bueno

12:30 *Assessing the performance of well-balanced WENO and TENO schemes for atmospheric flows*, Adrián Navas Montilla

12:50 *Esquemas de volúmenes finitos implícitos y semi-implícitos bien equilibrados de tipo Lagrangiano-Proyectado para las ecuaciones de aguas someras*, Celia Caballero Cárdenas

13:10 *Reduced-order models applied to hyperbolic transport equations*, Pablo Solán Fustero

Viernes 10 de Febrero
Moderadora: *Irene Gómez Bueno*

10:00 *Numerical evolution of the resistive relativistic magnetohydrodynamic equations: a minimally implicit Runge-Kutta scheme*, Samuel Santos Pérez

10:20 *Vertical discretizations in depth-averaged models for geophysical flows*, José Garres Díaz

10:40 *High-order fully well-balanced numerical methods for one-dimensional blood flow with discontinuous properties*, Ernesto Pimentel García

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Real Sociedad Matemática Española

León, Febrero de 2023

Esquemas de volúmenes finitos implícitos y semi-implícitos bien equilibrados de tipo Lagrangiano-Proyectado para las ecuaciones de aguas someras

C. Caballero-Cárdenas,* M. J. Castro* T. Morales de Luna* M. L. Muñoz-Ruiz*

En este trabajo presentamos aproximaciones numéricas para las ecuaciones de aguas someras basadas en la estrategia de volúmenes finitos de tipo Lagrangiano-Proyectado. Estos esquemas pueden interpretarse como un algoritmo de dos pasos que consiste en primero resolver las ecuaciones de aguas someras en coordenadas lagrangianas (lo que se conoce como paso lagrangiano) y después proyectar el resultado a coordenadas eulerianas (lo que se conoce como paso de proyección). El paso lagrangiano puede tratarse de manera implícita, mientras que la proyección siempre se hace explícitamente.

El uso de esta estrategia permite desacoplar los fenómenos acústicos y de transporte, de manera que se pueden desarrollar esquemas con pasos temporales grandes en los que la restricción CFL se basa en las ondas de transporte y no en las acústicas.

En este trabajo seguimos la estrategia descrita en [1, 3] para definir los esquemas de tipo Lagrangiano-Proyectados y [2] para asegurar su carácter bien equilibrado.

Agradecimientos Este trabajo está parcialmente apoyado por los proyectos RTI2018-096064-B-C21 y RTI2018-096064-B-C22 financiados por MCIN/AEI/10.13039/501100011033 y "FEDER Una forma de hacer Europa", proyectos P18-RT-3163 de la Junta de Andalucía y UMA18-FEDERJA-161 de la Junta de Andalucía-FEDER-Universidad de Málaga. C. Caballero-Cárdenas cuenta con el apoyo de la ayuda FPI2019/087773 financiada por MCIN/AEI/10.13039/501100011033 y "El FSE invierte en tu futuro".

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VI Congreso de Jóvenes Investigadores

Real Sociedad Matemática Española

León, Febrero de 2023

Vertical discretizations in depth-averaged models for geophysical flows

J. Garres-Díaz*

Many efforts have been devoted to the modelling and numerical simulation of geophysical flows, mainly due to their importance to develop early warning systems against natural hazards (tsunamis, floods, landslides, etc). Concretely, the Navier-Stokes equations describe the motion of fluids, and also other materials that flows. Nevertheless, huge computational efforts are needed to solve this system, making necessary other approaches.

A very popular strategy to reduce the computational efforts is to use the so-called Shallow Water models. In these models, vertical variations are neglected under the hypotheses that vertical effects are small when compared to the horizontal ones. Models derived under this assumption have been applied to many geophysical application since the pioneering work of [4]. Despite of the low computational cost associated of shallow models, their main drawback is the lack of vertical information in the simulations, which is a high price in many applications.

In this talk, which is based on the results in [2], we review some methods to overcome this loss of vertical structure in shallow flows. On the one hand, multilayer (or layer-averaged) models ([1]) split the vertical domain in virtual layers, applying the shallow water hypothesis inside each layer. On the other hand, moment methods ([3]) consider a polynomial expansion of the velocity profile along the vertical direction. It introduces some new unknowns in the system, that corresponds to the deviation from a constant vertical profile of velocity. Both approaches make possible to describe variations along the normal direction. Here, we introduce a common framework for vertical discretizations of Euler equations based on the σ-coordinates framework. It allows us to compare moment and multilayer models, and to proposed a new family of improved multilayer-moment methods.

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VI Congreso de Jóvenes Investigadores
Real Sociedad Matemática Española
León, Febrero de 2023

Una familia de esquemas de volúmenes finitos implícitos y semi-implícitos de alto orden bien equilibrados para sistemas de leyes de equilibrio unidimensionales

I. Gómez-Bueno [*], S. Boscarino [**] M.J. Castro[*], C. Parés[*] , G. Russo[**]

El objetivo de este trabajo es diseñar métodos numéricos implícitos y semi-implícitos de alto orden bien equilibrados para sistemas generales unidimensionales de leyes de equilibrio. Se considera la técnica introducida por dos de los autores en [1] para métodos explícitos, cuya clave es la aplicación de operadores de reconstrucción exactamente bien equilibrados. Dicha estrategia requiere conocer las soluciones estacionarias del sistema. Si no es el caso, se diseñan esquemas que son bien equilibrados siguiendo la metodología presentada en [2]-[3]. Este procedimiento se combina con un método de discretización temporal para la evolución del tiempo del tipo RK-IMEX o RK-implícito (véase [4]). La metodología se aplicará a varios sistemas de leyes de equilibrio, que van desde problemas escalares simples como la ecuación de Burgers hasta sistemas más complejos como las ecuaciones de aguas poco profundas con fricción de Manning.

Agradecimientos Este trabajo está parcialmente apoyado por el proyectos RTI2018-096064-B-C21 financiado por MCIN/AEI/10.13039/501100011033 y "FEDER Una forma de hacer Europa", proyecto P18-RT-3163 de la Junta de Andalucía y UMA18-FEDERJA-161 de la Junta de Andalucía-FEDER-Universidad de Málaga. C. Caballero-Cárdenas cuenta con el apoyo de la ayuda FPU2019/01541 financiada por MCIN/AEI/10.13039/501100011033 y "El FSE invierte en tu futuro".

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VI Congreso de Jóvenes Investigadores
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León, Febrero de 2023

Exactly well-balanced schemes for the shallow-water equations with Coriolis forces

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In this work we present two novel well-balanced high-order finite volume schemes for the shallow-water system with Coriolis source term.

We consider first the 1D shallow-water system with Coriolis term that could be written as a hyperbolic balance law: $\partial_t \mathbf{q} + \partial_x \mathbf{f}(\mathbf{q}) = \mathbf{s}(\mathbf{q})$, where $\mathbf{q} = (h, hu, hv)$, $\mathbf{f}(\mathbf{q}) = (hu, hu^2 + gh^2/2, huv)$ and $\mathbf{s}(\mathbf{q}) = (0, fhv - gh\partial_x z, -fhu)$. As usual, h denotes the water depth, u and v are the two components of the averaged horizontal velocity, z designates the topography, g is the constant gravitational acceleration and f the Coriolis parameter. The main objective is to define a high-order (≥ 3) exactly well-balanced finite volume scheme for the stationary solutions of the system. By exactly well-balanced we mean a method that is able to preserve the cell-averages, computed by a quadrature formula, of the exact stationary solutions of the system. Here we follow the general procedure described in [3], where the well-balanced property is achieved by combining a standard consistent first order numerical flux for the homogeneous system, with a exactly well-balanced reconstruction operator. In particular, the method proposed here extends the one presented in [4], where only first and second order schemes are proposed.

Next, we consider the 2D shallow-water system with a Coriolis source term that could be written as $\partial_t \mathbf{q} + \partial_x \mathbf{f}(\mathbf{q}) + \partial_y \mathbf{g}(\mathbf{q}) = \hat{\mathbf{s}}(\mathbf{q})$, where $\mathbf{g}(\mathbf{q}) = (hu, huv, hv^2 + gh^2/2)$ and $\hat{\mathbf{s}}(\mathbf{q}) = (0, -gh\partial_x z(x,y) + fhv, -gh\partial_y z(x,y) - fhu)$.

The design of general high-order exactly well-balanced finite volume schemes for 2D balance laws is harder than in 1D, as stationary solutions are now solutions of 2D nonlinear PDEs. In the literature, we can find different strategies to design well-balanced schemes for multi-dimensional problems, mainly in the framework of shallow flows, see for instance [1, 2]. In the framework of 2D shallow-water flows with Coriolis forces we have designed first and second order finite volume schemes that capture approximate stationary solutions corresponding to geostrophic equilibrium, that is, solutions that satisfy:$\nabla \cdot \mathbf{u} = 0$, $\partial_x \eta = fv/g$, $\partial_y \eta = -fu/g$, where $\eta = h + z$ is the free surface. This stationary solution is obtained as a limit on the shallow-water equations which results in an asymptotic behaviour of our scheme.

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High-order Implicit-Explicit scheme for the Exner model without gravity effects

E. Macca[*] M. J. Castro-Diaz[**] G. Russo[*]

The aim of this talk is introduce an Implicit-Explicit (IMEX) strategy to compute the Shallow water equation [1, 2] focusing on the sediment evolution in the Exner model for sediment transport [3] in Shallow Water framework and improve both stability and efficiency. The main difficulty in this model is the presence of several time scales. One associated with the temporal evolution of the sediment, generally very long with a much slower velocity; one related to the velocity of free-surface waves, generally very fast that implies an hard restriction in the time step; and one related to the velocity of the fluid with.

Unfortunately, as known, an explicit method implies a strong stability restriction due to the velocity of the free-surface waves. This restriction involves in a very long computation time that could be reduced neglecting the free-surface waves behaviour and looking directly at the sediment evolution. The objective is to drastically improve the efficiency in the computation of the evolution of the sediment by treating water waves implicitly, thus allowing much larger time steps than the one allowed by standard CFL condition on explicit schemes.

Agradecimientos Indam-project, GNCS 2022.

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León, Febrero de 2023

Assessing the performance of well-balanced WENO and TENO schemes for atmospheric flows

A. Navas-Montilla,* I. Echeverribar*

Numerical Weather Prediction (NWP) models have experimented an important transition in the last decade towards non-hydrostatic models. At the same time, the use of very high-order numerical schemes has been popularized, as they prove very efficient in modern High-Performance computing facilities and allow to achieve a high resolution in space and time [1]. This allows to obtain unprecedented resolution levels.

In this work, we present a family of very high order well-balanced numerical schemes for the computation of the compressible Euler equations with gravity source term, with application to meso- and micro-scale atmospheric flows. The method is based on augmented Riemann solvers, which allow to preserve the exact equilibrium between fluxes and source terms at cell interfaces. Three different spatial reconstruction methods are considered, the WENO [2], the TENO [3] and the linear reconstruction. The TENO method has been extensively used by the computational fluid dynamics community in recent years for the simulation of compressible turbulent flows, however, its application to NWP is still unexplored.

The WENO method uses a variable set of stencils where lower order polynomials are first constructed. Then, these lower order polynomials are combined either to create a higher order polynomial in smooth regions or an off-center reconstruction able to capture discontinuities in non-smooth regions. The definition of a smoothness indicator allows to distinguish between those two cases. On the other hand, the TENO method features an incremental stencil selection in an ENO-like manner, where one candidate stencil is either selected for the final reconstruction with the optimal weight, or discarded completely when crossed by discontinuities. According to [3], the TENO reconstruction ensures that the spectral properties and the accuracy order of the underlying linear scheme can be recovered up to intermediate wavenumber range.

We have examined the performance of the TENO scheme, in comparison with the WENO and linear reconstructions, for the resolution of different atmospheric phenomena involving the propagation of waves, discontinuous solutions and turbulent fields. We observe that the TENO reconstruction outperforms the WENO and linear reconstructions when considering discontinuous solutions and turbulence. For such cases, the WENO reconstruction adds excessive numerical diffusion, damping the small-scale eddies, whereas the linear reconstruction yields to oscillatory results. Only the TENO scheme provides a non-oscillatory solution while retaining a high accuracy in smooth regions.

Acknowledgements This work was funded by the Fundación Universitaria Antonio Gargallo under the project 2021/B010 (Navas-Montilla).

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VI Congreso de Jóvenes Investigadores

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León, Febrero de 2023

High-order fully well-balanced numerical methods for one-dimensional blood flow with discontinuous properties

E. Pimentel-García* L. O. Müler† E. F. Toro‡ C. Parés*

We are interested in the numerical study of the one-dimensional blood flow model with discontinuous mechanical and geometrical properties introduced in [1]. We present the mathematical model together with its nondimensional form.
In the context of hyperbolic systems of balance laws, it is well-known that well-balanced methods, i.e, methods that preserve all or some stationary solutions, are necessary to correctly capture the propagation of waves generated by small perturbations of a steady state but also to correctly capture the steady waves that stand on a discontinuity as we will see. We do an exhaustive investigation of all its stationary solutions and we propose high-order fully well-balanced numerical methods that are able to preserve all of them. They are based on the combination of the Generalized Hydrostatic Reconstruction (see [2]) and well-balanced reconstruction operators (see [3], [4]). These methods are able to deal with more than one discontinuous parameter. Several numerical tests are shown to prove its well-balanced and high-order properties, and its convergence to the exact solutions.

Acknowledgments The research of EPG and CP was partially supported by the Spanish Government(SG), the European Regional Development Fund(ERDF), the Regional Government of Andalusia(RGA), and the University of Málaga(UMA) through the projects of reference RTI2018-096064-B-C21 (SG-ERDF), UMA18-Federja-161 (RGA-ERDF-UMA), and P18-RT-3163 (RGA-ERDF). EPG was also financed by the European Union – NextGenerationEU.

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VI Congreso de Jóvenes Investigadores
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Numerical evolution of the resistive relativistic magnetohydrodynamic equations: a minimally implicit Runge-Kutta scheme

Samuel Santos-Pérez*, Isabel Cordero-Carrión*, Clara Martínez-Vidallach*

Significant magnetic fields are present in relevant astrophysical scenarios like active galaxies, relativistic jets, quasars, or compact objects. In general, physical resistivity has to be taken into account when modeling these scenarios and, therefore, a consistent treatment for the resistivity is necessary in numerical simulations.

We present the Minimally-Implicit Runge-Kutta (MIRK) methods for the numerical evolution of the resistive relativistic magnetohydrodynamic equations, following the approach proposed by [1] of an augmented system of evolution equations to numerically deal with constraints. These equations take the form

$$\begin{aligned} \partial_t E^j &= S_E^j - \sigma\, W\big(E^j + (v \times B)^j - (v_l E^l) v^j\big), \\ \partial_t Y &= S_Y, \end{aligned}$$

where E and B denote the electric and magnetic field and Y the rest of the evolved variables. S_E^j and S_Y are source terms, v the velocity and W the Lorentz factor. The term with the conductivity σ multiplying in the source of the first equation is potentially high and is called resistive or stiff term.

Schemes based on pure explicit methods fail when stiff terms are present, and other approaches rely on Implicit-Explicit (IMEX) Runge-Kutta schemes, see [2]. In general, compared to explicit schemes, IMEX methods need to apply the recovery (which can be very expensive computationally) of the primitive variables from the conserved ones in numerous additional times. Moreover, the use of an iterative process for the recovery could have potential convergence problems, increased by the additional number of required loops. In addition, the computational cost of the previous IMEX approach in comparison with the standard explicit methods is much higher.

The MIRK methods are able to deal with stiff terms producing stable numerical evolutions, minimize the number of recoveries needed in comparison with IMEX methods, their computational cost is similar to the standard explicit methods and can actually be easily implemented in numerical codes which previously used explicit schemes.

In this work first and second order MIRK methods are applied to two standard numerical tests: The self similar current sheet test and the circular polarized Alfvén waves test.

Acknowledgements The authors acknowledge support by the Spanish Agencia Estatal de Investigación / Ministerio de Ciencia, Innovación y Universidades through the Grants No. PGC2018-095984-B-I00 and PID2021-125458NB-C21, by the Generalitat Valenciana through the Grants No. PROMETEO/2019/071 and ACIF/2019/169 (European Social Fund) and by the Perimeter Institute for Theoretical Physics through the Simons Emmy Noether program.

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VI Congreso de Jóvenes Investigadores
Real Sociedad Matemática Española
León, Febrero de 2023

Reduced-order models applied to hyperbolic transport equations

P. Solán-Fustero* J.L. Gracia† A. Navas-Montilla* P. García-Navarro*

Hyperbolic equations play an important role in transport problems, such as heat transfer and free surface flows. The mathematical models used to solve those equations usually require a high computational cost due to the large spatial and temporal scales of the events. In recent times, reduced order models (ROM) based on the proper orthogonal decomposition (POD) are increasingly used to improve the computational efficiency of these problems [4, 5].

In this work, the ROM strategy is applied to the 1D linear advection equation. This equation is a very useful toy model to study the main features of this kind of problems $\frac{\partial u(x,t)}{\partial t} + \frac{\partial f(u,x,t)}{\partial x} = 0$.

The ROM strategy consists of two parts: the off-line part, in which the ROM is trained by applying the snapshot method [3] to the solutions of the full-order model (FOM) computed up to the training time; and the on-line part, in which the ROM is solved up to the same training time.

First, the FOM can be built following different methods and multiple orders of discretization, in space and in time [2]. And second, the ROM is developed by applying the Galerkin method [1] to the FOM, so each version of the FOM generates a different version of the ROM.

The main objective of this work is to study the performance of different combinations of ROM versions trained with different FOM schemes. In this way, we will use numerical methods such as the Lax-Friedrichs, the Lax-Wendroff and the Roe methods, which allows for different orders of discretisation.

The computational improvement of the ROM models with respect to their FOM counterparts is examined in terms of accuracy and computational time.

Acknowledgement The authors would like to express their gratitude to Dr. Donsub Rim for interesting discussions on reduced-order models. This work was funded by the Spanish Ministry of Science and Innovation under the research project PGC2018-094341-B-I00. This work has also been partially funded by Gobierno de Aragón through Fondo Social Europeo (T32-20R and E24-17R, Feder 2019-2021 "Construyendo Europa desde Aragón").

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Mathematical Challenges in Fluid Dynamics and Kinetic Theory

Organizadores: Diego Alonso, Claudia García, Eduardo García-Juárez y David Poyato

Lunes 6 de Febrero
Moderadores: *David Poyato y Eduardo García-Juarez*

12:30 *Monokineticity and mean-field limit for strongly singular Cucker-Smale model*, Michał Fabisiak.

13:00 *Homogenization of Stokes equation with randomly rotating particles*, Marta Leocata.

18:45 *On the Rayleigh-Taylor instability*, Björn Gebhard

19:15 *Long time behaviour of BGK models for heat transfer*, Josephine Evans

Martes 7 de Febrero
Moderadores: *Jaemin Park y Claudia García*

11:30 *Turbulence and convex integration*, Francisco Mengual

12:00 *Macroscopic limit of the Vlasov-Poisson-Fokker-Planck model*, Alain Blaustein

12:30 *Study of the asymptotic behavior of the NNLIF model*, Alejandro Ramos Lora

13:00 *Highest Cusped Waves for the Burgers-Hilbert and Fractional KdV equations*, Joel Dahne

18:45 *Small scale creation for the 2D Boussinesq Equation*, Jaemin Park

19:15 *Skeleton for the one-dimensional aggregation equation*, Juan Carlos Cantero

Miércoles 8 de Febrero
Moderador: *Diego Alonso Orán*

10:00 *Existence and decay for some fourth order problems*, Martina Magliocca

10:30 *Muskat equation with unbounded data*, Omar Sanchez Antonio

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León, Febrero de 2023

Macroscopic limit of the Vlasov-Poisson-Fokker-Planck model

A. Blaustein*

The Vlasov-Poisson-Fokker-Planck (VPFP) model describes a plasma where electrons are subjected to collisions with a fixed ion background and where they interact through electrostatic effects. In this talk, we will focus on the macroscopic limit of the VPFP model. We will present a recent work [1] where we propose a L^2 convergence result and strengthen previous results ([2, 3, 4, 5]) by obtaining explicit rates of convergence which hold without restriction on the size of the initial data nor on the phase-space dimension.

Agradecimientos Special thanks to David Poyato for inviting me to this conference.

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VI Congreso de Jóvenes Investigadores

Real Sociedad Matemática Española

León, Febrero de 2023

Skeleton for the one-dimensional aggregation equation

A. Juan Carlos Cantero*

Let N be the fundamental solution of the Laplacian. We consider the aggregation equation

$$\begin{cases} \rho_t + \operatorname{div}(\rho v) = 0, \\ v(\cdot, t) = -\nabla N * \rho(\cdot, t), \\ \rho(\cdot, 0) = \rho_0, \end{cases}$$

when the initial density is the characteristic function of some set Ω_0.

It is well known that the solution of this problem blows up at time 1. We study the limit measure at the blow up time whenever Ω_0 is either an open or a compact domain. We observe differences between the two situations.

We also inspect the limit set (which is called *skeleton*) where this measure is supported.

This is a joint work with Joan Orobitg (UAB).

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VI Congreso de Jóvenes Investigadores
Real Sociedad Matemática Española
León, Febrero de 2023

Highest Cusped Waves for the Burgers-Hilbert and Fractional KdV equations

Joel Dahne [*]

In this talk I'll present a proof of the existence of a periodic highest, cusped, traveling wave solution for the Burgers-Hilbert equation as well as ongoing work on extending the method used to a family of fractional KdV equations. The proof is to a large extent computer assisted and the focus will be on the reduction to a fixed point that lends itself to computer assisted tools as well as the basic ideas of the computer assisted parts.

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VI Congreso de Jóvenes Investigadores
Real Sociedad Matemática Española
León, Febrero de 2023

Long time behaviour of BGK models for heat transfer

Josephine Evans *

This is based on a joint work with Angeliki Menegaki from Institut Hautes Etudes Scientifique the paper is [1]. I will discuss the BGK equation which is a toy model designed to have similar properties as the Boltzmann equation for dilute gasses. Our particular setting is looking at the equation on a bounded domain with thermal boundary conditions. I will discuss the existence of steady states for this and related equations and if time permits I will discuss more recent progress on stability of these steady states.

Agradecimientos This work was supported in part by a FSMP postdoctoral fellowship and the ANR grant ANR-17-CE40-0030 then later by the Leverhulme early career fellowship ECF-2021-134.

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VI Congreso de Jóvenes Investigadores
Real Sociedad Matemática Española
León, Febrero de 2023

Monokineticity and mean-field limit for strongly singular Cucker-Smale model

A. Michał Fabisiak* B. Jan Peszek†

Cucker-Smale model, introduced in 2007 [1], describes the evolution of particles aligning their velocities according to nonlocal interaction protocol. On the particle level it reads

$$\begin{cases} \dot{x}_i = v_i, \\ \dot{v}_i = \dfrac{1}{N} \displaystyle\sum_{i \neq j = 1}^{N} \psi(|x_i - x_j|)(v_j - v_i), \\ (x_i(0), v_i(0)) = (x_{i0}, v_{i0}) \in \mathbb{R}^{2d}, \end{cases}$$

where psi is a so-called *communication weight* and it dictates the strength of the interaction between particles. We will focus on the strongly singular case of $\psi(s) = |s|^{-\alpha}$ with $\alpha \geq d$.

As $N \to \infty$, particle system is replaced by its kinetic counterpart, namely

$$\begin{cases} \partial_t \mu + v \cdot \nabla_x \mu + \mathrm{div}_v\,(F(\mu)\mu) = 0, \quad x \in \mathbb{R}^d, v \in \mathbb{R}^d \\ F(\mu)(t,x,v) := \displaystyle\int_{\mathbb{R}^{2d}} \psi(|x-y|)(w-v)\mu(t,y,w)\,\mathrm{d}w\,\mathrm{d}y. \end{cases}$$

We will consider weak, measure-valued solutions μ and see that under the assumptions that they are uniformly compactly supported and weakly continuous in time, they are monokinetic. This reduces our considerations to the macroscopic scale, known as the Euler-alignment system

$$\begin{cases} \partial_t \rho + \mathrm{div}_x(\rho u) = 0, \\ \partial_t(\rho u) + \mathrm{div}_x(\rho u \otimes u) = \Lambda_\rho u, \\ \Lambda_\rho u := \displaystyle\int_{\mathbb{R}^d} \psi(|x-y|)(u(y) - u(x))\rho(y)\rho(x)\,\mathrm{d}y. \end{cases} .$$

Consequently, this allows us to perform direct micro- to macroscopic mean field limit from the particle to Euler-alignment system and leads to our final result - existence of weak solutions to the fractional Euler-alignment system with almost arbitrary initial data in $\mathbb{R}^1$, including the possibility of vacuum. Existence can be extended to $\mathbb{R}^2$ under the a priori assumption that the density of the mean-field limit has no atoms. All the results are presented in [2].

Agradecimientos The work has been partly supported by the Polish National Science Centre's Grant No. 2018/31/D/ST1/02313 (SONATA). JP's work has been additionally supported by the Polish National Science Centre's Grant No. 2018/30/M/ST1/00340 (HARMONIA).

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Real Sociedad Matemática Española

León, Febrero de 2023

On the Rayleigh-Taylor instability

Björn Gebhard*

We consider the inhomgeneous incompressible Euler equations describing two fluids with different constant densities under the influence of gravity. Initially the fluids are supposed to be at rest and separated by a flat horizontal interface with the heavier fluid being on top of the lighter one. Due to gravity this configuration is unstable, the two fluids begin to mix in a more and more turbulent way. This is called the Rayleigh-Taylor instability. In the talk we will see how admissible solutions to the Euler equations reflecting a turbulent mixing of the two fluids in a quadratically growing zone can be constructed. Furthermore, if time allows, we will discuss an arising selection problem for the averaged motion of solutions.

This is based on joint works with József Kolumbán, László Székelyhidi, Jonas Hirsch.

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Homogenization of Stokes equation with randomly rotating particles

Marta Leocata [*], Richard Hofer [†], Amina Mecherbet [‡]

We consider suspensions of rod-like particles in a fluid. Our main aim is to understand the appearance of the stress term in the macroscopic fluid equation in the Doi model, through a microscopic description. Since the derivation of the full model is out of reach, we focus on a toy model where evolution in time of particles is frozen, the rotational motion of particles is decoupled from the fluid motion and the fluid motion is modeled by Stokes equation. Using the method of reflections, we derive the macroscpic model described above as the homogenization limit of many small particles (randomly rotating). The talk is based on the work [1].

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VI Congreso de Jóvenes Investigadores

Real Sociedad Matemática Española

León, Febrero de 2023

Existence and decay for some fourth order problems

R. Granero-Belinchón [*], M. Magliocca [†]

We will see some fourth order problems arising in Physics which model different processes, such as the growth of crystal surfaces and wetting-dewetting processes.
Mathematically speaking, we will focus on global existence and regularity results for problems as

$$\begin{cases} u_t = F(t, x, u, \nabla u, \dots, \Delta^2 u) & \text{in } [0,T] \times \mathbb{T}^N, \\ u(0,x) = u_0(x) & \text{in } \mathbb{T}^N, \end{cases}$$

where $\mathbb{T}^N = [-\pi, \pi]^N$ is the N–dimensional torus and the initial data u_0 belong to Wiener spaces. The particular choices of F will describe the model in object.

These results are contained in a joint work with R. Granero Belinchón [1], and in [2].

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León, Febrero de 2023

Turbulence and convex integration

Á. Castro * D. Faraco † F. Mengual ‡ L. Székelyhidi Jr. §

Despite turbulence can be experimented in everyday phenomena (e.g., moving a teaspoon in a cup of coffee) its rigorous mathematical description is still one of the biggest challenges in mathematical physics. In the last years, the convex integration method allowed to prove rigorously the Onsager conjecture [1, 2] on the anomalous dissipation of energy (zeroth law of turbulence). In this talk we shall see how this method allows to relate the dissipation of energy with the growth of the turbulence zone associated to the Kelvin-Helmholtz instability [3] and also the growth of the mixing zone associated to the Saffman-Taylor instability [4].

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Real Sociedad Matemática Española

León, Febrero de 2023

Small scale creation for the 2D Boussinesq Equation

Alexander Kiselev [*] Jaemin Park [†] Yao Yao [‡]

In this talk, we study long-time behaviors of the two-dimensional incompressible Boussinesq equations without thermal diffusion. While the 2D Boussinesq equations is known to possess global solutions with the presence of viscosity, it remains a outstanding open problem whether the inviscid case can exhibit a finite-time blow up. In the viscous case, we established algebraic growth of the Sobolev norms of the solutions for all time. For the inviscid case, we obtained the growth of the gradient of the temperature, assuming that the global solution exists for all time. The initial data under consideration in this work is not too restrictive. More precisely, we only require certain symmetry and sign conditions. The key ingredient of the proof is to derive a norm-inflation from the decay of an anisotropic Sobolev norm of the temperature, which can be observed in the conservation of energy. This work is a joint work with A. Kiselev and Y. Yao.

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Real Sociedad Matemática Española
León, Febrero de 2023

Study of the asymptotic behavior of the NNLIF model

M. J. Cáceres J. A. Cañizo A. Ramos Lora *

In the recent decades, a wide variety of studies on systems of Partial Differential Equations (PDE) have been developed to describe neural networks. Nonlinear Noisy Leaky Integrate and Fire neuronal (NNLIF) models are one of the simplest systems in this direction [1]. Previous works have investigated the existence of solutions to these systems, as well as the long-term behavior [2, 3, 4, 5].

Relying on the literature together with our own analytical and numerical work [6], we have deepened our work on the asymptotic behavior of the solutions to these equations. In this endeavor, our work is based on the study of the properties of the linear model and a new technique to understand the nonlinear model. In this direction, we have extended the previous knowledge on the existence of solutions to the linear model, as well as studied its regularization properties and used the existence of spectral gap for the semigroup associated with the linear equation.

Our research allows us to improve results published in recent years on the stability of the solutions of these equations, including some cases with several steady states.

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VI Congreso de Jóvenes Investigadores

Real Sociedad Matemática Española

León, Febrero de 2023

Muskat equation with Unbounded Data

Omar Sánchez*

We consider the Muskat equation with an unbounded initial data of the form

$$x^2 + g(x, 0).$$

We show existence of local solutions in Sobolev spaces $H^s(\mathbb{R})$ with $s \geq 3$. The method of proof relies in the study of a evolution equation for $g_t(x, t)$ in order to obtain suitable energy estimates.

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Análisis Matemático Aplicado y Ecuaciones Diferenciales

Organizadores: Diego Alonso, Rafael Granero y Martina Magliocca

Lunes 6 de Febrero
Moderadores: *Rafael Granero-Belinchón y Martina Magliocca*

12:30 *Uniform in gravity estimates for 2D water waves*, Siddhant Govardhan Agrawal.

13:00 *A limiting absorption principle for time-harmonic isotropic Maxwell and Dirac equation*, Lucrezia Cossetti.

18:45 *Traveling waves close to the Couette flow*, Daniel Lear

19:15 *On Variable Lebesgue Spaces and Navier-Stokes equations*, Gastón Vergara-Hermosilla

Martes 7 de Febrero
Moderadores: *Diego Alonso Orán y Daniel Lear*

10:00 *Free boundary dynamics of gravity Stokes flow*, Elena Salguero

10:30 *Quasiperiodic solutions in the Euler equation*, Francisco Torres Lizaur

11:30 *Rigorous thin film approximations of the one-phase unstable Muskat problem*, Edoardo Bocchi

12:00 *Spectral Analysis in Quantum Mechanics*, Naiara Arrizabalaga

12:30 *Keller estimates of the eigenvalues in the gap of Dirac operators*, Fabio Pizzichillo

13:00 *On the symmetry conjecture for eigenfunctions*, Ángel David Martínez Martínez

VI Congreso de Jóvenes Investigadores

Real Sociedad Matemática Española

León, Febrero de 2023

Uniform in gravity estimates for 2D water waves

Siddhant Agrawal*

We consider the 2D gravity water waves equation on an infinite domain. We prove that the time of existence of solutions is uniform even as the gravity parameter $g \to 0$. The energy estimate used to prove this is moreover scaling invariant with respect to all the scaling transformations of the Euler equation. The energy also allows for interfaces with angled crests and cusps and we also prove an existence result for $g = 0$.

As an application of this energy estimate, we then consider the water wave equation with no gravity where the fluid domain is homeomorphic to the disc. We prove a local wellposedness result which allows for interfaces with angled crests and cusps and then by a rigidity argument, we show that there exists initial interfaces with angled crests for which the energy blows up in finite time, thereby proving the optimality of this local wellposedness result.

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VI Congreso de Jóvenes Investigadores
Real Sociedad Matemática Española
León, Febrero de 2023

Spectral Analysis in Quantum Mechanics

Naiara Arrizabalaga [*]

The mathematical interest in Dirac operators has recently been renewed by the study of graphene in the physics literature. Graphene (modeled by a 2 dimensional Dirac operator) has proven, both from fundamental and technological points of view, to be a remarkable material due to its electronic properties. Though, the absence of a band gap structure in the energy spectrum of the charge carriers has some severe limitations in the possible technological applications.

On the other hand, the goal of one of the first physical papers considering Dirac operators on bounded domains [2] was to introduce a phenomenological model able to describe the confinement of quarks and the structure of hadrons (composite particles made of quarks). This work had a major impact on elementary particle Physics and the model became famously known as the MIT bag model.

Those are just a couple of examples of applications of the Dirac equation coupled with different potentials that justify the importance of their study. Moreover, the spectral analysis of those operators becomes essential, since in Quantum Mechanics the only observable objects are described by eigenvalues. Therefore, in this talk we will focus on some of the latest results related to the spectral analysis of the Dirac operator. We will see different potentials such as the Coulomb type one or others that are defined in the boundary of regular domain in the three dimensional space, like the delta-shell interactions. We will focus mainly in the last ones, particularly in the MIT bag model. We will study the eigenvalue curves, their asymtotics and a shape optimization problem [1].

This is a joint work with Albert Mas, Tomas Sanz-Perela and Luis Vega.

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VI Congreso de Jóvenes Investigadores

Real Sociedad Matemática Española

León, Febrero de 2023

Rigorous thin film approximations of the one-phase unstable Muskat problem

Edoardo Bocchi* Francisco Gancedo†

In this talk we address the one-phase Muskat problem driven by gravity and surface tension. The regime considered is unstable with the fluid on top of a dry region. By a novel approach using a depth-averaged formulation, we derive two asymptotic approximations for this scenario. The lower order approximation is the classical thin film equation, while the higher order approximation provides a new refined thin film equation. We prove the optimal order of convergence in the shallowness parameter to the original Muskat solutions for both models with low-regular initial data.

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A limiting absorption principle for time-harmonic isotropic Maxwell and Dirac equation

L. Cossetti*

In this talk we investigate the $L^p - L^q$ mapping properties of the resolvent associated with the time-harmonic isotropic Maxwell and Dirac operator. As spectral parameters close to the spectrum are also covered by our analysis, we establish a $L^p - L^q$ type limiting absorption principle for these operators. Our analysis relies on new results for Helmholtz systems with zero order non-Hermitian perturbations. The talk is based on a joint work with R. Mandel and on an ongoing project with R. Mandel and R. Schippa.

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VI Congreso de Jóvenes Investigadores

Real Sociedad Matemática Española

León, Febrero de 2023

On Variable Lebesgue Spaces and Navier-Stokes equations

Gaston Vergara-Hermosilla* Diego Chamorro†

In this talk we will discuss the uniqueness of Leray solutions of the stationary Navier-Stokes equations in the whole space $\mathbb{R}^3$. Under some additional hypotheses, stated in terms of variable Lebesgue spaces $L^{p(\cdot)}(\mathbb{R}^3)$, we will show that the trivial solution $\vec{u}$ is the unique solution. On the other hand, we will present a generalisation of the Kato theory of mild solutions for the Navier-Stokes equations in terms $L^{p(\cdot)}(\mathbb{R}^3)$.

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VI Congreso de Jóvenes Investigadores
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Traveling waves close to the Couette flow

Ángel Castro,[*] Daniel Lear [†]

In this talk we present the existence of a large family of new, nontrivial and smooth traveling waves for the 2D Euler equation at an arbitrarily small distance from the Couette flow in H^s, with $s < 3/2$, at the level of the vorticity. The speed of these waves is of order 1 with respect to this distance. This result strongly contrasts with the setting of very high regularity in Gevrey spaces (see [1]), where the problem exhibits an inviscid damping mechanism that leads to relaxation of perturbations back to nearby shear flows. It also complements the fact that there not exist nontrivial traveling waves in the $H^{\frac{3}{2}+}$ neighborhoods of Couette flow (see [2]).

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VI Congreso de Jóvenes Investigadores
Real Sociedad Matemática Española
León, Febrero de 2023

Quasiperiodic solutions in the Euler equation

Francisco Javier Torres de Lizaur

Consider an N dimensional torus $\mathbb{T}^N$ embedded in the infinite dimensional space of smooth divergence-free vector fields on some domain, and with the property that the solutions of the Euler equation with any initial velocity u_0 in $\mathbb{T}^N$ remain there for all time, defining an ODE on $\mathbb{T}^N$. I will review recent results [1, 2] on the existence of this type of invariant tori of fluid velocities, and on their dynamics. This is joint work with Alberto Enciso and Daniel Peralta-Salas.

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VI Congreso de Jóvenes Investigadores
Real Sociedad Matemática Española
León, Febrero de 2023

On the symmetry conjecture for eigenfunctions

Ángel D. Martínez *

In this talk we will provide a brief introduction to the eigenfunctions of the Laplace-Beltrami operator, enough to state the so-called symmetry conjecture. Then we will present certain results (positive and negative) obtained in collaboration with F. Torres de Lizaur. Time permitting we will comment on further results, work in progress or open questions.

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Real Sociedad Matemática Española

León, Febrero de 2023

Free boundary dynamics of gravity Stokes flow

Elena Salguero*

The dynamics of viscous waves is a classical and active subject in the analysis of PDEs arising from fluid dynamics. The fluids are typically subject to different forces, such as surface tension or gravity forces, depending on the physical phenomena that one wants to capture. The dynamics also depends on the geometry of the problem (for instance, note that smooth solutions to the Navier-Stokes problem are well-known in 2D, but the 3D scenario remains a longstanding open problem).

At very low Reynolds number, where the viscous forces are dominant, the dynamics of the fluids is described by the Stokes equation. There is a vast literature about the study of the Stokes operator and Stokes-based models. Motivated by the previous works and applicability, in joint work with F. Gancedo and R. Granero-Belinchón [1], we have analyzed the dynamics of two incompressible, viscous and immiscible fluids filling the horizontally periodic strip. We consider the fluids to have different densities and to be subject to gravity forces. This induces the dynamics of the free boundary between the fluids.

In this talk, we will study the derivation of the previous model using the contour dynamics approach and we will answer fundamental questions such as the well-posedness and stability of the free boundary in the stable and unstable regime of the densities.

Acknowledgments E. Salguero was partially supported by the grant PRE2018-083984, funded by MCIN/AEI/ 10.13039/501100011033, by the ERC through the Starting Grant H2020-EU.1.1.-639227, by the MICINN (Spain) through the grants EUR2020-112271 and PID2020-114703GB-I00, and by the Junta de Andalucía through the grant P20-00566.

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Recent Advances in Local and Non-local PDE's

Organizadores: Ángel Arroyo, Juan Carlos Felipe y Tomás Sanz

Miércoles 8 de Febrero
Moderador: *Ángel Arroyo*

10:00 *Concentration versus Simplification in Aggregation-Diffusion Equations*, David Gómez-Castro

10:30 *Concentration phenomena arising in Aggregation Fast-Diffusion equations*, Alejandro Fernández-Jiménez

11:30 *A sharp Hardy inequality in fractional Sobolev spaces*, Francesca Bianchi

12:00 *Multiple solutions for the nonlocal Liouville equation in* $\mathbb{R}$, Antonio J. Fernández

12:30 *Bound and ground states for a family of singular systems in* $\mathbb{R}^N$, Rafael López-Soriano

13:00 *Bernstein method for nonlocal operators*, Marvin Weidner

Jueves 9 de Febrero
Moderadores: *Juan-Carlos Felipe-Navarro y Tomás Sanz-Perela*

10:00 *Nonlocal concave-convex critical problems with mixed boundary conditions*, Alejandro Ortega

10:30 *Positive solutions to critical competitive systems in low dimension*, María Medina

11:30 *Self-similar solutions for the generalized surface quasi-geostrophic equation*, Claudia García

12:00 *An interior regularity result for the MEMS problem*, Renzo Bruera

12:30 *Global regularity of stable solutions to semilinear equations*, Íñigo U. Erneta

13:00 *A new class of double phase variable exponent problems and a Nehari manifold approach*, Ángel Crespo-Blanco

17:00 *Evolution driven by the infinity fractional Laplacian*, Félix del Teso

17:30 *The KPP diffusion-reaction problem in the hyperbolic space*, Irene Gonzálvez

18:00 *Blowing up solutions to elliptic problems with natural growth in the gradient*, Salvador López

18:30 *Generic regularity of free boundaries for the thin obstacle problem*, Damià Torres

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A sharp Hardy inequality in fractional Sobolev spaces

F. Bianchi*

We consider the fractional Hardy inequality and we look for the sharp constant on open subsets of the Euclidean space. In particular we compute such a constant in the class of convex sets under the restrictions $s\,p \geq 1$ or $p = 2$.

Some of the results presented have been obtained in collaboration with Lorenzo Brasco (University of Ferrara), Firoj Sk (Okinawa Institute of Science and Technology) and Anna Chiara Zagati (University of Parma).

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VI Congreso de Jóvenes Investigadores
Real Sociedad Matemática Española
León, Febrero de 2023

An interior regularity result for the MEMS problem

Renzo Bruera *

In this talk we present an interior regularity result for the class of stable solutions to a semilinear elliptic equation with a singular nonlinearity. More precisely, given a bounded open set $\Omega \subset \mathbb{R}^n$, we consider the problem

$$\begin{cases} -\Delta u = f(u) & \text{in } \Omega \\ 0 < u < 1 & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega, \end{cases} \tag{1}$$

where the nonlinearity $f \in C^1((0,1))$ is assumed to be positive, nondecreasing, and to satisfy $f(1) = +\infty$ and $\int_0^1 f(t)dt = +\infty$. The model nonlinearities for this problem are the powers, i.e., $f(t) = (1-t)^{-p}$ for $p > 1$. For $p = 2$ and $n = 2$ this equation models the deflection of a dielectric elastic membrane in a microelectromechanical system (MEMS).

A solution u to problem (1) is said to be stable if the following inequality holds:

$$\int f'(u)\xi^2 \leq \int |\nabla \xi|^2, \quad \forall \xi \in C_c^\infty(\Omega). \tag{2}$$

Under a Crandall-Rabinowitz type assumption on f, we are able to prove that $u < 1$ in Ω up to the optimal dimension $n = 6$, and as a consequence, u is smooth in Ω (in contrast to singular solutions, which attain the value 1 somewhere in Ω).

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León, Febrero de 2023

A new class of double phase variable exponent problems and a Nehari manifold approach

Á. Crespo-Blanco*

During the last decade the so-called double phase operator has drawn attention from researchers. Originally it was introduced by Zhikov in the context of homogenization and elasticity theory (see [7]) and as an example for the Lavrentiev phenomenon (see [6]). It regained popularity after some novel regularity results for local minimizers of the corresponding functional (see [1, 2, 3]).

In this talk we introduce a new class of quasilinear elliptic equations driven by the so-called double phase operator with variable exponents. This part of the talk is based on [4], where we prove useful properties of the corresponding Musielak-Orlicz Sobolev spaces (an equivalent norm, uniform convexity, Radon-Riesz property with respect to the modular, density of smooth functions) and also properties of this new double phase operator (continuity, strict monotonicity, (S+)-property). In contrast to the previously known constant exponent case we are able to weaken the assumptions on the data.

After that we consider a problem with superlinear right-hand side. This last part of the talk is based on [5], in which under very general assumptions on the nonlinearity we prove a multiplicity result for such problems, whereby we show the existence of a positive solution, a negative one and a solution with changing sign. The sign-changing solution is obtained via the Nehari manifold approach and, in addition, we can also give information on its nodal domains. Furthermore, we derive a priori estimates on the solutions in the L^∞-norm under the very general setting used above.

Acknowledgments Ángel Crespo-Blanco was funded by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) under Germany's Excellence Strategy - The Berlin Mathematics Research Center MATH+ and the Berlin Mathematical School (BMS) (EXC-2046/1, project ID:
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VI Congreso de Jóvenes Investigadores
Real Sociedad Matemática Española
León, Febrero de 2023

Global regularity of stable solutions to semilinear equations

Iñigo U. Erneta*†

In this talk, I will announce some recent results on the global regularity of stable solutions to semilinear equations. These are a follow-up to the interior estimates in [2].

Given a bounded domain $\Omega \subset \mathbb{R}^n$ of class $C^{1,1}$, we consider stable solutions to $a_{ij}(x)u_{ij} + b_i(x)u_i + f(u) = 0$ in Ω vanishing on the boundary. Here, stability amounts to the nonnegativity of the principal eigenvalue of the linearized equation. We will show that stable solutions are globally Hölder continuous when $n \leq 9$. This dimension is optimal, since there are examples of unbounded stable solutions for $n \geq 10$. Before our work, the only optimal result [1] involved the model operator, the Laplacian, and needed a C^3 regularity assumption on the domain.

To prove the global result, flattening out the boundary, it will suffice to establish a priori estimates on half-balls. Our bounds there will be independent of the nonlinearity $f \in C^1$, which we assume to be nonnegative, nondecreasing, and convex. For the extension of these estimates to $C^{1,1}$ domains, it is essential to make the constants in our bounds depend on specific norms of the coefficients, namely, the $C^{0,1}$ norm of a_{ij} and the L^∞ norm of b_i.

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VI Congreso de Jóvenes Investigadores

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León, Febrero de 2023

Multiple solutions for the nonlocal Liouville equation in $\mathbb{R}$

Antonio J. Fernández *

We construct multiple solutions to the Liouville type equation

$$(-\Delta)^{\frac{1}{2}} u = K(x) e^{u} \quad \text{in } \mathbb{R}. \tag{1}$$

More precisely, for K of the form $K(x) = 1 + \varepsilon\kappa(x)$ with $\varepsilon \in (0,1)$ small and $\kappa \in C^{1,\alpha}(\mathbb{R}) \cap L^{\infty}(\mathbb{R})$ for some $\alpha > 0$, we prove existence of multiple solutions to (1) bifurcating from the so-called *Aubin-Talenti bubbles*. These solutions provide examples of flat metrics in the half-plane with prescribed geodesic curvature $K(x)$ on its boundary. Furthermore, they imply the existence of multiple ground state soliton solutions for the Calogero-Moser derivative NLS.

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VI Congreso de Jóvenes Investigadores
Real Sociedad Matemática Española
León, Febrero de 2023

Concentration phenomena arising in Aggregation Fast-Diffusion equations

Alejandro Fernández-Jiménez*

Over the last two decades, a lot of attention has been given to the Aggregation-Diffusion equation

$$\partial_t \rho = \operatorname{div}\left(\rho \nabla (U'(\rho) + V + W * \rho)\right).$$

This family of equations has become popular because, amongst others, it models the mean-field limit of systems with a large number of interacting particles arising in biology. Because of that, there has been a lot of interest concerning the study of the asymptotic behaviour as $t \to \infty$, discussing the existence or not of Dirac deltas, which are usually described as concentration phenomena.

We can understand this problem as a gradient flow and this approach has lead to several satisfactory results [1]. However, the case in which we are interested, V and W regular enough and Fast Diffusion, $U(\rho) = \rho^m/(m-1)$ with $0 < m < 1$, presents a challenge because the free energy associated is no longer convex and a gradient flow approach does not guarantee convergence to the minimiser. Therefore, in order to overcome these difficulties we follow a similar approach to the one in [2], where they study the case $W = 0$, V regular enough and Fast Diffusion. Then, we rely on a combination of gradient flows, viscosity solutions and compactness arguments to study the asymptotic behaviour of the problem. We will review some of the literature and we will show how to use these techniques to obtain this type of results.

The talk presents ongoing joint work with Prof. J. A. Carrillo and Prof. D. Gómez-Castro.

Acknowledgements This research was supported by the Advanced Grant Nonlocal-CPD (Nonlocal PDEs for Complex Particle Dynamics: Phase Transitions, Patterns and Synchronization) of the European Research Council Executive Agency (ERC) under the European Union's Horizon 2020 research and innovation programme (grant agreement No. 883363).

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Real Sociedad Matemática Española

León, Febrero de 2023

Self-similar solutions for the generalized surface quasi-geostrophic equation

C. García*

In this talk, we will construct a large class of non-trivial (non radial) self-similar solutions of the generalized surface quasi-geostrophic equation. To the best of our knowledge, this is the first rigorous construction of any self-similar solution for these equations. Moreover, the solutions are of spiral type, locally integrable and may have a change of sign. This is a joint work with Javier Gómez-Serrano.

Acknowledgements C. García has been partially supported by the European Research Council ERC-StG-852741 (CAPA), the MINECO–Feder (Spain) research grant number RTI2018–098850–B–I00, and the Junta de Andalucía (Spain) Project FQM 954.

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León, Febrero de 2023

Concentration versus Simplification in Aggregation-Diffusion Equations

D. Gómez-Castro*

Over the last two decades, intense work has been devoted to the Aggregation-Diffusion equation

$$\partial_t \rho = \operatorname{div}\Big(\rho \nabla (U'(\rho) + V + W * \rho)\Big).$$

This family of problem model, amongst other phenomena, the mean-field limit of systems with a large number of interacting particles arising in biology. It includes, for example, the famous model by Keller-Segel for chemotaxis.

From the mathematical point of view, they benefit from having a gradient flow structure. There is a long literature discussing the existence of gradient-flow solutions, characterising the existence/non-existence of minimisers of the associated free-energy functional and, in particular, discussing the existence or not of delta Deltas. The presence of a Delta is usually described as a concentration phenomena. In the absence of candidate stationary state, the solutions diffuse. We will discuss some of the key elements of the theory.

The aim of this talk is to present two works at opposite ends of the spectrum. On the one hand, the asymptotic formation of Dirac deltas in the case of Fast Diffusion [1]. On the other, a general result of asymptotic simplification to the heat kernel when W is bounded, with suitably integrable derivatives [2].

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VI Congreso de Jóvenes Investigadores

Real Sociedad Matemática Española

León, Febrero de 2023

The KPP diffusion-reaction problem in the hyperbolic space.

*I. Gonzálvez**

In this talk we study the Cauchy problem in the hyperbolic space for the heat equation with KPP type forcing term. Considering the isometries and the geodesics of this space, we work with some initial data trapped between two invariant isometric sets. This allows us to prove how the solution converges asymptotically to some specific travelling wave in a moving frame with a logarithmic correction. Our results translate the known asymptotic theory of the KPP problem in the Euclidean space to the Hyperbolic space paying special attention to the geometry of this space.

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VI Congreso de Jóvenes Investigadores
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León, Febrero de 2023

Blowing up solutions to elliptic problems with natural growth in the gradient

S. López-Martínez*

In the talk we will discuss the following model:

$$\begin{cases} -\Delta u = \lambda u + \mu(x)|\nabla u|^q + f(x), & x \in \Omega, \\ u = 0, & x \in \partial\Omega. \end{cases}$$

Here, Ω is a bounded domain of $\mathbb{R}^N$ with smooth boundary, $\lambda \in \mathbb{R}$, $1 < q \leq 2$, μ is a non-negative bounded function and f is a non-negative $L^p(\Omega)$ function for $p > N$.

In this general framework, the problem is nowadays well-understood for $\lambda < 0$. More precisely, there exists a unique bounded weak solution for every $\lambda < 0$, and the family of solutions in terms of λ constitutes a continuum (see [1] and references therein). Moreover, if the problem does not admit a solution for $\lambda = 0$, then the continuum blows up as $\lambda \to 0^-$ and the asymptotic behavior is characterized in terms of the so-called ergodic problem (see [4]).

If, on the contrary, there is a solution for $\lambda = 0$, then the behavior of the branch of solutions for $\lambda \geq 0$ is not completely understood. It is conjectured that the branch bifurcates from infinity still at zero, meaning that there is a continuum of blowing up solutions as $\lambda \to 0^+$. However, this conjecture has been proven only in particular cases where, typically, restrictions depending on q, N or μ appear (see [3] and references therein).

In the talk we will give an overview of the results that are known for the range $\lambda > 0$. We will also present some new results in this direction as part of the joint work [2] with J. Carmona and P.J. Martínez–Aparicio.

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VI Congreso de Jóvenes Investigadores

Real Sociedad Matemática Española

León, Febrero de 2023

Bound and ground states for a family of singular systems in $\mathbb{R}^N$

R. López-Soriano*

In this talk we will focus on the solvability of a family of nonlinear elliptic systems defined in $\mathbb{R}^N$. Such equations contain Hardy type potentials and criticalities in Sobolev or Hardy-Sobolev sense coupled by a possible critical term. By means of variational techniques, we shall find ground and bound states in terms of the coupling parameter ν and the order of the different parameters and exponents. Actually, for a wide range of parameters we find solutions as minimzers or Mountain-Pass critical points of the energy functional on the underlying Nehari manifold.

Acknowledgments R.L-S. is currently supported by the grant Juan de la Cierva Incorporación fellowship (JC2020-046123-I), funded by MCIN/AEI/10.13039/501100011033, and by the European Union Next Generation EU/PRTR. He is also partially supported by Grant PID2021-122122NB-I00 funded by MCIN/AEI/ 10.13039/501100011033 and by "ERDF A way of making Europe".

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VI Congreso de Jóvenes Investigadores
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Positive solutions to critical competitive systems in low dimension.

María Medina*

We will analyze the existence and the structure of different sign-changing solutions to the Yamabe equation in the whole space and we will use them to find positive solutions to critical competitive systems in dimension 3 and 4.

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León, Febrero de 2023

Nonlocal Concave-Convex Critical problems with Mixed Boundary Conditions

Alejandro Ortega*

In this talk we introduce some existence results for nonlocal critical problems with concave-convex nonlinearities,

$$(P_\lambda) \qquad \left\{\begin{array}{rll} (-\Delta)^s u = \lambda u^q + u^{2_s^*-1}, & u > 0 & \text{in } \Omega, \\ B(u) = 0 & & \text{on } \partial\Omega, \end{array}\right.$$

with $\frac{1}{2} < s < 1$ and $0 < q < 2_s^* - 1$, $q \neq 1$, being $2_s^* = \frac{2N}{N-2s}$; $\lambda > 0$ and $\Omega \subset \mathbb{R}^N$, $N > 2s$, is a smooth bounded domain with mixed Dirichlet-Neumann boundary conditions

$$B(u) = u\chi_{\Sigma_{\mathcal{D}}} + \frac{\partial u}{\partial \nu}\chi_{\Sigma_{\mathcal{N}}},$$

where χ_A is the characteristic function of a set A, $\Sigma_{\mathcal{D}}$ and $\Sigma_{\mathcal{N}}$ are smooth $(N-1)$-dimensional submanifolds of $\partial\Omega$ such that $\Sigma_{\mathcal{D}}$ is closed with measure $|\Sigma_{\mathcal{D}}| = \alpha \in (0, |\partial\Omega|)$; $\Sigma_{\mathcal{D}} \cap \Sigma_{\mathcal{N}} = \emptyset$, $\Sigma_{\mathcal{D}} \cup \Sigma_{\mathcal{N}} = \partial\Omega$ and $\Sigma_{\mathcal{D}} \cap \overline{\Sigma}_{\mathcal{N}} = \Gamma$ is a smooth $(N-2)$-dimensional submanifold.

Using variational and topological methods we prove:

Theorem 0.1 *Let $0 < q < 1$ and $N > 2s$. Then, there exists $0 < \Lambda < \infty$ such that the problem (P_λ)*

1. *has no solution for $\lambda > \Lambda$,*
2. *has a minimal solution for any $0 < \lambda < \Lambda$. Moreover, the family of minimal solutions is increasing in λ,*
3. *has at least one solution for $\lambda = \Lambda$,*
4. *has at least two solutions for $0 < \lambda < \Lambda$.*

Theorem 0.2 *Let $1 < q < 2_s^* - 1$ and $N > 2s\left(1 + \frac{1}{q}\right)$. The problem (P_λ) has at least one solution for any $\lambda > 0$.*

The existence of a second solution in Theorem 0.1 crucially relies on a Strong Maximum Principle for mixed fractional problems that will be also discussed in the talk. In particular, let u be the solution to

$$\left\{\begin{array}{rl} (-\Delta)^s u = f & \text{in } \Omega, \\ B(u) = 0 & \text{on } \partial\Omega, \end{array}\right.$$

with $f \in L^\infty(\Omega)$, $f \gneq 0$ and v be the solution to

$$\left\{\begin{array}{rl} (-\Delta)^s v = g & \text{in } \Omega, \\ B(u) = 0 & \text{on } \partial\Omega, \end{array}\right.$$

with $g \in L^p(\Omega)$, $p > \frac{N}{s}$ and $g \gneq 0$. Then, the following holds.

Theorem 0.3 *There exists a constant $C > 0$ such that*

$$\left\| \frac{v}{u} \right\|_{L^\infty(\Omega)} \leq C\|g\|_{L^p(\Omega)}.$$

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Evolution driven by the infinity fractional Laplacian

Félix del Teso*

We consider the evolution problem associated to the following infinity fractional Laplacian operator Δ_∞^s introduced by Bjorland, Caffarelli and Figalli as the infinitesimal generator of a non-Brownian tug-of-war game:

$$\sup_{|y|=1} \inf_{|\tilde{y}|=1} \int_0^\infty (\phi(x+\eta y) + \phi(x-\eta\tilde{y}) - 2\phi(x)) \frac{\mathrm{d}\eta}{\eta^{1+2s}}.$$

We first construct a class of viscosity solutions of the initial-value problem for bounded and uniformly continuous data. An important result is the equivalence of the nonlinear operator in higher dimensions with the one-dimensional fractional Laplacian when it is applied to radially symmetric and monotone functions. Thanks to this and a comparison theorem between classical and viscosity solutions, we are able to establish a global Harnack inequality that, in particular, explains the long-time behavior of the solutions. Finally, we propose a fully discrete and monotone finite-difference scheme, and support our theoretical results with numerical evidence.

The talk will be based on results appeared in [1, 2].

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VI Congreso de Jóvenes Investigadores

Real Sociedad Matemática Española

León, Febrero de 2023

Generic regularity of free boundaries for the thin obstacle problem

Damià Torres-Latorre*

The Signorini problem is a free boundary problem arising from elastostatics, and can also be seen as the counterpart to the obstacle problem when the obstacle has codimension 1. The main questions are understanding the regularity of solutions and the regularity of the free boundary.

In the first part of the talk, I will present the problem and discuss some relevant known results. Then, I will explain the concept of generic regularity in free boundary problems.

Finally, I will expose my recent result with Fernández-Real about generic regularity in the Signorini problem, and comment briefly on the recent developments that have made it possible.

Acknowledgements D.T. has received funding from the European Research Council (ERC) under the Grant Agreement No 801867 and from the grant RED2018-102650-T funded by MCIN/AEI/10.13039/501100011033.

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Bernstein method for nonlocal operators

Marvin Weidner *

The Bernstein method allows to prove derivative estimates for solutions to a large class of elliptic equations by application of the maximum principle to certain suitable auxiliary functions. In this talk, we explain how the Bernstein method can be extended to a large class of integro-differential equations driven by nonlocal operators that are comparable to the fractional Laplacian. Moreover, we discuss several applications of this technique to nonlocal obstacle problems in bounded domains, as well as to fully nonlinear integro-differential equations. This talk is based on a joint work with Xavier Ros-Oton and Damià Torres-Latorre.

Agradecimientos Marvin Weidner has received funding from the European Research Council (ERC) under the Grant Agreement No 801867

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Avances Recientes en Teoría de Control

Organizadores: Jon Asier Bárcena-Petisco y Diego A. Souza

Jueves 9 de Febrero
Moderador: *Diego A. Souza*

10:00 *Uncertainty principles in control theory of PDEs*, Ivan Moyano

10:30 *An optimal boundary control problem related to the Navier-Stokes equations in dimension 2*, Irene Marín-Gayte

Viernes 10 de Febrero
Moderadores: *Diego A. Souza y Jon Asier Bárcena Petisco*

10:00 *On the global controllability of the viscous Boussinesq system*, Kévin Le Balc'h

10:30 *Insensitizing control for the Boussinesq system using Carleman estimates*, Jefferson Prada

11:30 *Game theory in mathematical ecology*, Domènec Ruiz i Balet

12:00 *Control of advection-diffusion equations on networks and singular limits*, Nicola de Nitti

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León, Febrero de 2023

Control of advection-diffusion equations on networks and singular limits

J. A. Bárcena-Petisco* M. Cavalcante† G. M. Coclite ‡ N. De Nitti § E. Zuazua¶

We consider advection-diffusion equations posed on a tree-shaped networks with suitable transmission conditions at the junctions.

Using a control which is localized on the exterior nodes, we prove that the system is null-controllable. Moreover, we study the asymptotic behavior of the cost of the null-controllability as the diffusivity parameter vanishes: we show that it decays for a sufficiently large time and explodes for short times with an exponential rate.

These results have been proved in [1] and extend the ones previously obtained e.g. in [2, 3] (on a bounded domain in $\mathbb{R}^N$) to networks.

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VI Congreso de Jóvenes Investigadores

Real Sociedad Matemática Española

León, Febrero de 2023

On the global controllability of the viscous Boussinesq system

Kévin Le Balc'h*

In this talk, we focus on the small-time global controllability of the Boussinesq system, given a time $T > 0$ (possibly very small), given an initial data (possibly very large), the goal is to know if it is possible to steer the solution to the Boussinesq system at zero at time T. The control force is modeled by the choice of the values of the velocity field and the temper- ature on a (small) part on the boundary of the domain (2D or 3D). On the other parts, we assume that the velocity field and the temperature satisfy respectively a Navier slip-with-friction boundary condition and a Robin boundary condition. The strat- egy of the proof is crucially inspired by [1] dealing with the Navier-Stokes case. The main difficulties are the treatment of the boundary layers that appear near the uncontrolled boundary and the thermal effects. This is a joint work with F. W. Chaves- Silva, E. Fernández-Cara, J. L. F. Machado, D. A. Souza [2].

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VI Congreso de Jóvenes Investigadores

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León, Febrero de 2023

An optimal boundary control problem related to the Navier-Stokes equations in dimension 2

I. Marín-Gayte*, J.Tiago †

In this talk, we are going to analyze an optimal control problem related to the Navier-Stokes equations with Dirichlet-type conditions in one part of the boundary and Neuman-type conditions in another part. Splitting the boundary into portions on which different boundary conditions are set is rather common in several fluid dynamics applications. Dirichlet conditions are used for the problem at hand to impose a velocity profile on the inflow boundary and on the control boundary, and a no-slip condition at the lumen boundary, since it is assumed that there fluid particles perfectly adhere to the vessel wall. Neumann conditions prescribe a force per unit area as the normal component of the stress tensor; if the force is vanishing, the Neumann boundary portion is a free outflow – this is indeed the condition we assign on the outflow boundary. Note also that when Dirichlet boundary conditions are specified on the entire boundary the pressure is not uniquely defined. Furthermore, we assume that the control acts on the boundary with Dirichlet-type conditions, so that we understand our optimal control problem as a boundary control problem.

Boundary control problem with Dirichlet-Neuman conditions have been analyzed in [3], in the case of nonstationary Navier-Stokes equations, in two dimensions and in [4], considering instead non-stationary Navier-Stokes equations; however, it is considered a pure Dirichlet problem. In [1] the optimal boundary control existence analysis for Navier-Stokes equations with mixed boundary conditions is performed in three dimensions, without deriving a set of optimality conditions. Regarding the case of non-stationary Navier-Stokes equations, a frontier optimization problem similar to the one studied in our work has been analyzed in [2], but it is a pure Dirichlet problem.

So, we provide a comprehensive theoretical framework to address the analysis of the optimal control problem related to this system and the derivation of a system of first-order optimality conditions that characterizes the solution of the control problem. We take advantage of an adequate treatment of the control and frontier Dirichlet data carried out through the study of the reduced functional (functional that only depends on the control). Despite the fact that this approach is quite common, a detailed analysis for mixed boundary conditions is still lacking. We consider the minimization of a quadratic functional that includes the control and the state. Next, several algorithms are shown that compute both the minimum and the optimality system (gradient-type algorithms).

Agradecimientos The authors are partially support by Hemocontrol,PTDC/MAT-APL/7076/2020.

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VI Congreso de Jóvenes Investigadores
Real Sociedad Matemática Española
León, Febrero de 2023

Uncertainty principles in control theory of PDEs

Ivan Moyano*

In this talk we review some classical and recent results relating the uncertainty principles for the Laplacian with the controllability and stabilisation of some linear PDEs. The uncertainty principles for the Fourier transforms state that a square integrable function cannot be both localised in frequency and space without being zero, and this can be further quantified resulting in unique continuation inequalities in the phase spaces. Applying these ideas to the spectrum of the Laplacian on a compact Riemannian manifold, Lebeau and Robbaino obtained their celebrated result on the exact controllability of the heat equation in arbitrarily small time. The relevant quantitative uncertainty principles known as spectral inequalities in the literature can be adapted to a number of different operators, including the Laplace-Beltami operator associated to C^1 metrics or some Schödinger operators with long-range potentials, as we have shown in recent results in collaboration with Gilles Lebeau (Nice) in [1] and Nicolas Burq (Orsay) in [2] and [3], with a significant relaxation on the localisation in space. As a consequence, we obtain a number of corollaries on the decay rate of damped waves with rough dampings, the simultaneous controllability of heat equations with different boundary conditions and the controllability of the heat equation with rough controls.

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VI Congreso de Jóvenes Investigadores
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Insensitizing control for the Boussinesq system using Carleman estimates

Jefferson Prada *

In this work we prove the existence of insensitizing controls for a viscous newtonian fluid wherein thermic effects are taken into account, the so called Bousinesq system. This system consists of a Navier-Stokes system with a heat equation. The novelty is that we insensitize the rotational of the solution of the Navier Stokes system together the gradient of the solution of the heat equation, using controls with two component fixed at zero. This problem can be formulated as a null controllability problem for a nonlinear cascade system for which we follow the usual duality approach. First, we prove a suitable Carleman inequality for a system coupling, which leads to the null controllability at any positive time. Finally, we deduce a local null controllability result for the cascade system by a local inverse argument.

Agradecimientos This study is supported by “Beca Doctorado Nacional 2020" of Agencia Nacional de Investigación y Desarrollo (ANID) in Chile,“Beca de Incentivo a la Iniciación Científica 2022" and “Beca de Asistencia a Congresos 2022-2" of Universidad Técnica Federico Santa María.

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Real Sociedad Matemática Española

León, Febrero de 2023

Game theory in mathematical ecology

Domènec Ruiz-Balet*

The tragedy of the commons is a concept in economic theory introduced by William Forster Lloyd that states the following:

The selfish action of individuals that have access to a common resource can lead to the depletion of the resource do to the uncoordinated action of the individuals. This is in contrast with the common management of the resource or when regulations are applied to avoid this behavior.

One can understand this concept mathematically as the existence of particular Nash equilibria that lead to the depletion of the resource, in contrast with a common optimization approach.

This is the main objective of this talk, to show that in population dynamics this situation can actually happen in the most simple spatially dependent mathematical models such as bistable reaction-diffusion equations in the context of mean-field games and monostalbe ones in the case of N-player games.

We will see that in the N-player games for the monostable equation Nash equilibria tend to have a very particular structure numerically, which can be understood under certain regimes in the sense of rearrangements.

This new type of mean-field games will be introduced as well as the notion in which we will understand the tragedy of the commons.

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Resúmenes - Pósters

En la sesión de pósters se presentarán los siguientes trabajos:

- *Subvariedades homogéneas minimales en los espacios hiperbólicos complejos*, por Ángel Cidre Díaz
- *Quantum Efficient Algorithms To Found Substructures On Finite Algebras*, por Jefferson Miguel Hernández Cáceres
- *Explaining how pseudo-multiband images improve semantic segmentation for U-Net in outdoor agricultural environments*, por Clara Isabel López-González
- *Generalized Hilbert operators acting on classical spaces of analytic functions*, por Elena de la Rosa

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Real Sociedad Matemática Española

León, Febrero de 2023

Subvariedades homogéneas minimales en los espacios hiperbólicos complejos

Ángel Cidre Díaz *

Los espacios hiperbólicos son variedades de Riemann no compactas con un alto grado de simetría, lo que geométricamente se traduce en que son espacios simétricos. En particular, los espacios simétricos son también homogéneos, lo cual quiere decir que su grupo de isometrías actúa transitivamente sobre el espacio. Esto último motiva el estudio de las subvariedades homogéneas, es decir, las órbitas generadas por subgrupos del grupo de isometrías del espacio ambiente. Más concretamente, es natural considerar las órbitas homogéneas resolubles, esto es, las que provienen del conocido como modelo resoluble del espacio hiperbólico, el cual además dota naturalmente a la variedad de una estructura de grupo de Lie [1]. Por otro lado, las subvariedades minimales, aquellas para las que el campo curvatura media es nulo, surgen como una generalización a cualquier dimensión del concepto de geodésica. En los espacios hiperbólicos reales se sabe que las órbitas homogéneas minimales son totalmente geodésicas [2], pero para el resto de espacios hiperbólicos aún no se conocen exactamente. De este modo, en este trabajo nos centramos en clasificar las subvariedades homogéneas minimales resolubles de los espacios hiperbólicos complejos, que son los espacios hiperbólicos más sencillos para los cuales no se había obtenido una clasificación explícita.

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Quantum Efficient Algorithms To Found Substructures On Finite Algebras

J.M. Hernández Cáceres [*] I.F.Rúa [†] Elías F. Combarro [‡]

Quantum computing is based on quantum phenomena, and it is well-known that some quantum algorithms outperform their classical counterparts. For instance, Simon's algorithm [**?**] was one of the first quantum algorithms to show an exponential speed-up versus the best classical algorithm in solving a specific problem. Moreover, based on the quantum Fourier transform (perhaps one of the most important unitary transformation in quantum computing) factoring integers can be solved in polynomial time by Shor's algorithm [**?**], while none classical algorithm with such a complexity is known. These algorithms can be regarded under the framework of the hidden subgroup problem (HSP).

Definition. *Let G be a group and $H \subseteq G$ one of its subgroups. Let S be any set and $g : G \to S$ be a function that distinguishes cosets of H, i.e., for all $g_1, g_2 \in G$, $f(g_1) = f(g_2) \Leftrightarrow g_1H = g_2H$. The hidden subgroup problem (HSP) is to determine a generating set for the subgroup H.*

A well-known theorem [[**?**], Theorem 5.1] tell us that a classical computer must make $\Omega(\sqrt{N})$ queries to solve the HSP, where N is the number of subgroups whose only common element is the identity. However, for the abelian case, efficient quantum algorithm solving the HSP are known. Now, let K be a finite field, let A be a non-associative and non-commutative K-algebra with a fixed basis $\beta = \{e_1, \ldots, e_n\}$, and consider the additive group $(A, +)$, i.e., the elements of the algebra with the addition operation. Given two elements $a, b \in A$, we define the commutator $[a, b] = ab - ba$. We define the associator, as a multilinear map $[\cdot, \cdot, \cdot] : A \times A \times A \to A$ given by $[x, y, z] = (xy)z - x(yz)$. Consider the following sets, known as right nucleus, middle nucleus, left nucleus, nucleus and center:

$$\begin{aligned}
N_r(A) &= \{a \in A : [x, y, a] = 0 \text{ for all } x, y \in A\} \\
N_m(A) &= \{a \in A : [x, a, y] = 0 \text{ for all } x, y \in A\} \\
N_l(A) &= \{a \in A : [a, x, y] = 0 \text{ for all } x, y \in A\} \\
N(A) &= N_r(A) \cap N_m(A) \cap N_l(A) \\
Z(A) &= N(A) \cap \{a \in A : [a, x] = 0 \text{ for all } x \in A\}
\end{aligned}$$

Finding those sets can be stated in terms of the HSP, and it is important in the context of the classification of finite semifields, see for instance [**?**] and [**?**]. We explicity and efficiently construct quantum circuits that, from the structure constants of the algebra, can be used to determine these sets in (quantum) polynomial time.

Acknowledgment This work has been partially supported by PAPI-21-PF-10 and by the Spanish Government, under Projects PID2021-123461NB-C22.

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VI Congreso de Jóvenes Investigadores

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León, Febrero de 2023

Explaining how pseudo-multiband images improve semantic segmentation for U-Net in outdoor agricultural environments

Clara I. López-González* María J. Gómez-Silva† Eva Besada-Portas† Gonzalo Pajares‡

The difficulty of semantic segmentation tasks in the outdoor agricultural domain, due to the changing enviromental conditions, has led to increasing the use of deep neural networks, which have proven remarkable performance. Along with the desired generalization ability, these models carry a lack of interpretability and explainability due to their black box nature. In order to gain a better understanding of how neural networks learn, what information is important for decision making, or whether that information is being correctly coded, explainable methods have been developed under the E*x*plainable *A*rtificial *I*ntelligence (*x*AI) framework [1]. It ranges from simple visualizations through optimization to using topological data analysis [2]. However, this type of network analysis is not so common in the agricultural imaginery, and if it exists, it does not usually go beyond traditional well-known methods.

In this work, the deep learning U-Net model is used for agriculture image semantic segmentation. The crop row benchmark presented in [3] is used to train two models: a) one with the original dataset and b) another one with the pre-processed dataset. The pre-processing is done by adding 5 pseudo-bands built from the RGB bands computing different vegetation indices, widely used in agricultural image segmentation. The main objective is to explain why and how these pseudo-multiband images improve model performance against the original RGB images, and to interprate and compare how both neural models learn. To achieve this, we analyze the layers of the two networks using the gradient class activation map (Grad-CAM) [4]. A novel explainable approach is developed by comparing the topology of latent spaces with persistence landscapes on the first homology group. The results provide a detailed explanation of how pseudo-multiband images improve agricultural semantic segmentation.

Acknowledgements This work has been supported by the Research Project IA-GES-BLOOM-CM (Y2020/TCS-6420) of the Synergic program of the Comunidad Autónoma de Madrid and by the Research Project AMPBAS (RTI2018-098962-BC21) of the National Societal Challenges program of the Spanish Ministry of Science, Innovation and Universities. The research by Clara I. López- González is supported by a FPU Ph.D. scholarship from the Spanish Ministry of Universities.

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Real Sociedad Matemática Española

León, Febrero de 2023

Generalized Hilbert operators acting on classical spaces of analytic functions

E. de la Rosa *

In this poster, we consider the Hilbert-type operator defined by

$$H_\omega(f)(z) = \int_0^1 f(t) \left(\frac{1}{z} \int_0^z B_t^\omega(u)\, du \right) \omega(t) dt,$$

where $\{B_\zeta^\omega\}_{\zeta\in\mathbb{D}}$ are the reproducing kernels of the Bergman space A^2_ω induced by a radial weight ω in the unit disc $\mathbb{D}$. Observe that the choice $\omega = 1$ gives the integral representation of the classical Hilbert operator

$$H(f)(z) = \int_0^1 f(t) \frac{1}{1-tz} dt,$$

therefore we are interested in the study of the features of a radial weight ω so that H_ω has some of the nice properties of the (classical) Hilbert operator.

We show that H_ω is bounded on the Hardy space H^p, $1 < p < \infty$, if and only if

$$\sup_{0\leqslant r<1} \frac{\widehat{\omega}(r)}{\widehat{\omega}\left(\frac{1+r}{2}\right)} < \infty, \tag{1}$$

and

$$\sup_{0<r<1} \left(\int_0^r \frac{1}{\widehat{\omega}(t)^p} dt \right)^{\frac{1}{p}} \left(\int_r^1 \left(\frac{\widehat{\omega}(t)}{1-t} \right)^{p'} dt \right)^{\frac{1}{p'}} < \infty,$$

where $\widehat{\omega}(r) = \int_r^1 \omega(s)\, ds$.

We also prove that $H_\omega : H^1 \to H^1$ is bounded if and only if (1) holds and

$$\sup_{r\in[0,1)} \frac{\widehat{\omega}(r)}{1-r} \left(\int_0^r \frac{ds}{\widehat{\omega}(s)} \right) < \infty.$$

As for the case $p = \infty$, H_ω is bounded from H^∞ to BMOA, or to the Bloch space, if and only if (1) holds.

We also consider the action of H_ω on some spaces of analytic functions closely related to Hardy spaces.

These results are the consequence of joint work with N. Merchán and J. A. Peláez ([1] and [2]).

References

[1] J. A. Peláez, E. de la Rosa. *Hilbert-type operator induced by radial weight*. J. Math. Anal. Appl **495** (2021), no. 1, Paper No. 124689, 22 pp.

[2] N. Merchán, J. A. Peláez, E. de la Rosa. *Hilbert-type operator induced by radial weight on Hardy spaces*, submitted preprint
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