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Maximal averages along variable manifolds

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Maximal functions are of fundamental importance in Harmonic Analysis due to its close connection with almost everywhere convergence questions. A central example is the Hardy–Littlewood maximal function which given a non-negative locally integrable function f and any point $x \in \mathbb{R}^d$ outputs the largest average of f over all balls centered at x. The Lebesgue space mapping properties of this operator imply then the almost everywhere convergence of such averages to $f(x)$ as the radius of the balls shrink to 0 for $f \in L^p(\mathbb{R}^d)$ if $1 \leq p \leq \infty$.

In 1976, Stein [1] introduced the spherical maximal function, in which the averages are taken over spheres as opposed to balls, and established that for $d \geq 3$ this is bounded on $L^p(\mathbb{R}^d)$ if and only if $p > \frac{d}{d-1}$; moreover, a.e. convergence results related to solutions of the wave equation followed. The 2-dimensional case, which corresponds to the most singular situation, was established 10 years later by Bourgain [2]. Stein and Bourgain's seminal work initiated the study of maximal functions associated to other types of singular averages. In these types of questions, the curvature of the underlying manifolds plays a key role.

In this talk we present two extensions of Bourgain's result, in two different directions. The first one is a 3-dimensional counterpart, in which the averages are taken along dilates of curves in $\mathbb{R}^3$ with non-vanishing curvature and torsion, such as the helix. We prove that the corresponding maximal function is bounded on $L^p(\mathbb{R}^3)$ if and only if $p > 3$. The second one is 2-dimensional, and it is associated to a family of variable singular planar curves that originate from considering the Heisenberg group analogue of Bourgain's circular maximal function (which is 3-dimensional) and restricting it to Heisenberg radial functions. We prove that such a maximal function is bounded on $L^p(\mathbb{H}^1)$ on the class of Heisenberg radial functions if and only if $p > 2$.

If time permits, we will also discuss variation norm analogues and regularity properties of such maximal functions.

The talk will be mostly based on joint works with Shaoming Guo, Jonathan Hickman and Andreas Seeger [3, 4].

References

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